Deceptive Products and Competition in Search Markets*

Preliminary

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Abstract

We study a search market where firms have a choice between offering either efficient, high quality ("candid") products or inefficient, low quality ("deceptive") products which some ("naive") consumers fail to recognize as such. We derive an equilibrium in which both business models co-exist and show that as search frictions vanish, high quality goods are entirely driven out of the market. We show that market share and price dynamics can be non–monotone in search frictions, and we argue that while policy interventions that reduce search frictions such as the standardization of price and package formats may harm welfare, a price floor regulation and a minimum quality standard can improve welfare.

Keywords: Deceptive product, Inferior product, Naivete, Consumer Search

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1 Introduction

An important feature in many consumer markets is "product complexity" in the sense that fully understanding a product's features and assessing all consumption consequences requires technical, scientific, legal, or other expertise. This provides sellers of complex products with the opportunity to strategically frame and package their offers in ways that mislead consumers who lack this expertise. An important example are financial services whose understanding requires both financial literacy and an awareness of numerous future contingencies. Standard financial services such as mutual investment funds or credit cards are fraught with contract clauses and contingent fees hidden in fine print. As argued in Gabaix and Laibson (2006) and Armstrong and Vickers (2012), it is often difficult to make sense of such contract terms in traditional models of consumer behavior, as they generate enormous profits at the expense of few vulnerable consumers.[1] Another example are investment products composed of complex derivatives. Before the outbreak of the financial crisis, investment banks pushed the sale of seemingly top-rated, yet in truth toxic papers to ill-advised investors who were not aware of the risks they were exposed to.[2] Other examples of complex products comprise grocery products whose features (ingredients, health effects, origin, "organic" vs. "non–organic"), which are often prominently advertised, are difficult to ascertain for lay consumers, or consumer electronics where future usage costs are difficult to estimate.

In this paper, we analyze a market for a complex product when consumers need to engage in costly search to detect the price and the fit of a product.[3] We capture the link between product complexity and deception by assuming that some ("naive") consumers can be deceived about the true product quality. That is, firms have the choice of offering a "candid" product of high quality or a "deceptive" product of low quality which naive consumers, however, fail to recognize as such.[4]

[1] For example, Armstrong and Vickers (2012) discuss insufficient fund charges that made up 30% of the total revenue generated by current accounts in the year 2006, however, were only incurred by 23% of all customers according to the Office of Fair Trading (2008).

[2] As evidence of this, Deutsche Bank just recently agreed to pay a \$3.1 billion penalty and to provide relief to American consumers valued at \$4.1 billion in order to resolve a federal investigation of its sale of toxic mortgage backed securities.

[3] As documented by Line and Wildenbeest (2015) for Medigap insurance, search is likely to be a key factor in markets for insurance and financial services.

[4] Next to lack of expertise, there might be other, possibly complementary, reasons for consumer naivete which are widely accepted in the literature by now: limited attention, overconfidence in the own ability to assess products,

Candid products, while more costly to produce, are assumed to be socially efficient. As a result, in the benchmark without any naive consumers, competition ensures that firms only offer candid products.[5]

In contrast, in this paper we will show that when some consumers are naive, there is an equilibrium which displays market segmentation where the deceptive and the candid business model co–exist in the market. This equilibrium has some remarkable positive and normative comparative statics properties. As one of our main insights, we show that more intense competition in the form of lower search costs may have the detrimental effect that it increases the share of deceptive products in the market. Strikingly, in the limit, as search frictions get small, candid products are even entirely driven out of the market.[6] We will also discuss policy interventions that may, or may not, improve welfare. Among others, we will argue that policy measures that aim at facilitating price comparisons such as the imposition of standardized price or package format may be socially undesirable, and that a price floor regulation and a minimum quality standard may improve welfare in our setting.

Our finding that smaller search frictions may promote the share of deceptive products is consistent with the empirical evidence in Ellison and Ellison (2009) who report for the online market for memory modules that as a response to the better search technology (in particular for prices), firms began to "bundle low-quality goods with unattractive contractual terms, like providing no warranty and charging a 20 % restocking fee on all returns".[7]

We study market segmentation equilibria with the property that sophisticated consumers, who see through deception, never buy deceptive products but search until they find a suitable candid product. In contrast, naive consumer, who perceive all firms as homogeneous and fail to infer quality from price, purchase from the first firm they visit provided its price is in an acceptable

overconfidence in the comprehensiveness of regulation, or vulnerability to persuasive sales techniques, to name a few.

[5]In the absence of naive consumers, our model essentially corresponds to Wolinsky (1986) or Anderson and Renault (1999) where sellers compete in horizontally differentiated products, and consumers have to engage in costly search to observe price and product fit.

[6]Away from the limit, on the other hand, market share dynamics as well as prices may be non-monotone in search costs.

[7]Ellison and Ellison (2009) conclude that "given the variety of terms we observed, it would seem unwise to purchase a product without reading the fine print."

range that makes continued search unviable. As we show, this implies that deceptive and candid firms charge similar prices in equilibrium, and since candid quality is more costly to produce, their product choice is driven by the trade-off between selling the candid product at a low mark–up to all consumers and selling the deceptive product at a high mark–up to only naive consumers. Intuitively, market segmentation arises in equilibrium because naive consumers perceive all firms as homogeneous, and so their demand is inelastic as in Diamond (1971). Hence, deceptive firms can charge positive mark–ups ensuring strictly positive profits. In contrast, since candid firms compete for sophisticated consumers who compare prices, the mark–up of a candid firm depends on the number of candid competitors in the market. Therefore, for given search costs, one business model cannot exist alone, because if there are predominantly deceptive firms in the market, then offering a candid product gets relatively more profitable, as it attracts large demand from sophisticated consumers who are divided up between very few candid firms. Reversely, if there are predominantly candid firms in the market, deception can be seen as a way to avoid the fierce competition in the candid segment.

Our first comparative statics result states that in the limit, as search costs get small, candid firms are entirely driven out of the market. Thus, if only a small fraction of consumers is vulnerable to deception, intense competition in the form of small search costs has the striking consequence that the market will supply only deceptive quality. The basic intuition is that, as search costs get small, sophisticated consumers can compare firms essentially for free. This intensifies competition in the candid segment and eliminates a candid firm's mark–up and profit, ultimately leading all firms to adopt the deceptive business model which guarantees positive profits.[8]

While our limit result suggests that the number of candid firms in the market is increasing in search costs, this holds only under additional conditions. In fact, we show by an example that the fraction of candid firms may also decline in search costs. When this is the case, the price a candid firm charges may actually decrease with search costs. Similarly to standard search models, as search costs increase, sophisticated consumers search less, allowing candid firms, all else equal, to increase their price. In our framework, there is, however, an additional and novel indirect

[8]As we explain in the main text in detail, this intuition is somewhat incomplete because what matters for sophisticated consumers is not nominal, but effective search costs, that is, the costs it takes to find not any, but a candid firm. As we show, effective search costs are monotone in nominal search costs and, in fact, go to zero if nominal search costs do.

effect because the number of candid firms is endogenous. As we show, if the number of candid firms drops, sophisticated consumer spend more time searching for a suitable candid product. In effect, there are more sophisticated consumers present in the market and a single candid firm's demand gets more elastic, pushing its price down. Hence, if the fraction of candid firms decreases in search costs, this force works against the familiar effect that prices increase as consumers search less.[9]

Our comparative statics results suggest that changes in search costs may have non–standard welfare effects in our setting. In fact, we show that total welfare may increase in search costs. This is the case if the share of candid firms increases in search costs and the efficiency gains from a candid product are sufficiently large. Since prices are a wash from the perspective of total welfare, efficiency gains may then outweigh the higher search costs consumers incur. The ensuing welfare gains will, however, be unequally distributed among the market participants: Sophisticated consumers lose out because they are harmed by higher prices and higher search costs, whereas firms gain, and the effect on naive consumer welfare depends on whether the average quality increase offsets the price increase.

In the case in which welfare increases in search costs, policy interventions that aim at reducing search costs are undesirable. This may include policies that facilitate the comparability of products, such as requiring firms to use standardized price, package, and product information formats, or the promotion of online marketplaces that facilitate the exchange of price information. Another policy measure that is often discussed in the context of complex products are information and education campaigns that create consumer awareness (such as promoting financial literacy in the context of financial service markets). In our framework, this amounts to reducing consumer naivete. As we show, reducing the number of naive consumers is beneficial for both naive and sophisticated consumer welfare. The reason is that with a shrinking naive customer base, fewer firms adopt the deceptive business model. This not only reduces the effective search costs sophisticated consumer incur on average, but also creates, under additional assumptions, more price pressure in the candid segment. In this sense, naive consumers exert a negative externality

[9] We also study the effects of changes in the number of naive consumers on market shares and prices. Whereas the share of candid firm increases in the number of naive consumers as expected, prices may actually decrease. This comes somewhat surprisingly given that the demand of naive consumers is inelastic, and is also due to the fact that the share of candid firms is endogeneous.

on sophisticated consumers in our framework.[10]

In our setting, price competition becomes dysfunctional because naive consumers constitute a safe profit haven for firms: the tighter the margins in the candid segment, the higher the incentives to adopt the deceptive business model which guarantees positive profits. This suggests that impeding price competition by imposing a price floor might be beneficial in our framework. To address this point, we consider the effect of an exogenously imposed (marginal) price increase relative to the equilibrium price. As we show, this always improves total welfare because it makes offering candid products more profitable. Despite the price increase, a price floor regulation may even increase consumer welfare if the ensuing increase in the number of candid firms is sufficiently large.

Our paper is related to a literature in behavioral industrial organization which studies how firms (can) exploit boundedly rational consumers. Most closely related are papers which study markets where firms offer goods with (unavoidable) future add-on services which naive consumers fail to anticipate. We contribute to this literature by adding consumer search to the picture, allowing entirely novel dimensions of comparative statics, in particular with respect to search frictions. More specifically, in Gabaix and Laibson (2006), Armstrong and Vickers (2012) and Heidhues et al. (2017) deception occurs when firms engage in hidden add–on pricing, possibly leaving naive consumers with utility below their outside option. While in our framework, a deceptive good can be interpreted as a good with hidden unavoidable add-on costs, firms cannot price or disclose (unshroud) the add-on.[11] In the latter respect, our model is similar to Armstrong and Vickers (2012) which focuses on add-on pricing alone and finds that all firms generically either engage in or abstain from deceptive add-on pricing in equilibrium. In contrast, a key point of our paper is that the deceptive and non-deceptive business model co-exist. Our finding that sophisticated consumers are harmed by the presence of naive ones stands in contrast to the opposite finding in Gabaix and Laibson (2006) and Armstrong and Vickers (2012), but is consistent with Heidhues et al. (2017) where deceptive equilibria exist only if there are enough naive consumers

[10]This is in contrast to models where firms exploit naive consumers and use the proceeds to compete for sophisticated consumers (see, e.g., Gabaix and Laibson (2006), Armstrong and Vickers (2012).)

[11]In an extension to our basic model, we discuss the possibility that firms can unshroud, that is, turn naive into sophisticated consumers, and we show that in equilibrium neither deceptive nor candid firms have incentives to do so.

in which case sophisticates abstain from the market. In contrast, in our setting, sophisticates still consume, but due to the presence of naive consumers, have to search longer and pay higher prices. A final, rather striking, difference between Heidhues et al. (2017)'s and us is that the existence of an (exogenous) price floor is the root cause for equilibrium deception in their work, whereas in our setting, a price floor regulation can mitigate deception.

A closely related paper is also Piccione and Spiegler (2012) whose analysis, like ours, implies that policy interventions that aim at facilitating the comparability of products, for example by mandating standardized price formats, can turn out to harm consumer welfare. The underlying reason is, however, rather different. In Piccione and Spiegler (2012), firms choose price and pricing formats which determine whether consumers can compare prices across firms. Firms face a trade-off between maximizing sales volume by combining low prices with easily comparable formats and maximizing mark–ups by combining high prices with onerously comparable formats. Piccione and Spiegler (2012) identify conditions under which ("local") interventions that make relatively easily comparable formats even more easily comparable may backfire because the intervention makes the strategy which combines high prices with onerous comparability more profitable at the margin. As a result, prices will increase on average. If, on the other hand, the intervention is "global" and makes all formats more easily comparable, then Piccione and Spiegler (2012) show that the intervention is desirable. In contrast, in our model, prices may drop if they become more easily comparable, and what makes the intervention undesirable is that the quality in the market deteriorates, suggesting competing testable predictions. Also, in our framework, interventions that ease comparability (i.e., reduction in search costs) are "global" by definition. Finally, the underlying consumer bias is different. Whereas consumers in Piccione and Spiegler (2012) have difficulties in comparing prices, they have difficulties to assess a product's quality in our setting, suggesting different domains of applicability.[12]

Moreover, our paper contributes to the industrial organization literature on consumer search

[12]To the extent that seemingly pro–competitive interventions may backfire, our paper is also related to Varian (1980)'s model of sales where firms set prices so as to trade off attracting consumers who compare prices across firms and reaping loyal customers who do not compare prices. An intervention which leads to a higher number of competitors may backfire because it makes reaping loyal consumer relatively more profitable, entailing an average price increase. Also in Vickers et al. (2009), the seemingly consumer-friendly introduction of a price cap may actually be harmful such that consumers pay higher prices on average, as it reduces their incentives to search.

by integrating naive consumers and seller deception into the seminal papers by Wolinsky (1986) and Anderson and Renault (1999). Our result that prices may increase as search costs fall has appeared before in the literature but for different reasons, among others in Janssen and Moraga-González (2004), Armstrong and Zhou (2011), Janssen and Shelgia (2015), and Moraga-González et al. (2017).[13] In our case, it originates in the fact that the share of candid firms is endogenous which affects both effective search costs and the composition of demand for an individual firm. Somewhat similarly, in Moraga-González et al. (2017), lower search costs alter the composition of demand, but in their case, this is due to the market entry of less versed consumers whose demand is less elastic. The connection between the share of candid firms and effective search costs is somewhat reminiscent of Eliaz and Spiegler (2011) where a search engine operator may encourage low-relevance advertisers to enter the search pool in order to raise effective search costs and hence advertisers' profits with the goal to extract some of these profits with fees.[14] The theme that consumers are unaware of what kind of products they purchase also appears in Gamp (2016) who considers consumer who trade off the risks of a bad buy with the savings on search costs, so that it is a rational consumer's decision to remain uninformed. In contrast to the current paper, Gamp (2016) focuses on the pricing and obfuscation strategies of firms under market conditions rather than on their product choice.

This paper is organized as follows. The next section introduces the model. Section 3 derives equilibrium conditions. Section 4 and 5 perform comparative statics, and section 6 discusses policy implications. Section 7 discusses extensions, and section 8 concludes. All proofs are in the appendix.

2 Model

We consider a search market with a unit mass of firms indexed by $k \in [0,1]$ and a unit mass of consumers indexed by $i \in [0,1]$. Each consumer seeks to purchase at most one good. Goods are differentiated, and consumer i's utility from purchasing the good offered by firm k is equal to

$$u_{ik} = q_k + \theta_{ik} - p_k, \tag{1}$$

[13] See also Cachon et al. (2008) and Choi et al. (2017).

[14] Similarly, in Ireland (2007) sellers posit multiple prices to reduce the effectiveness of consumer price search.

where q_k is the "quality" offered, and p_k is the price chosen by firm k. The term θ_{ik} is a consumer-firm specific match-value which represents idiosyncratic product fit. It is common knowledge that for all i, k, the associated random variable θ_{ik} is distributed on the support $[\underline{\theta}, \overline{\theta}]$ with (sufficiently smooth) cumulative density function F and mean zero (hence $\underline{\theta} < 0$), independent and identical across consumers and firms. We assume that the corresponding probability density function f is log-concave with $f(\overline{\theta}) > 0$ which implies that F has an increasing and unbounded hazard rate

$$h(\theta) \quad = \quad \frac{f(\theta)}{1-F(\theta)}. \tag{2}$$

The objective of our analysis is to study market outcomes when some consumers lack the ability to assess products and can be deceived about the true product characteristics. To capture this, we assume that firms can offer either a "candid good" of high quality $\overline{q}$ or a "deceptive good" of low quality $\underline{q} < \overline{q}$. The low quality product, however, is disguised (due to packaging, fine print, sales techniques, ...) so that some consumers cannot distinguish between candid and deceptive products. The quality of a product determines its production costs with $c(q)$ being the marginal costs for producing a good with quality q. Let $\underline{c} = c(\overline{q})$ and $\overline{c} = c(\overline{q})$, with $\underline{c} < \overline{c}$. Moreover, let $\Delta c = \overline{c} - \underline{c}$ and $\Delta q = \overline{q} - \underline{q}$. To make the problem interesting, we assume that the candid good is socially efficient,

$$\Delta q > \Delta c, \tag{3}$$

and that consumers, net of prices, prefer a candid over a deceptive product even if the former displays the worst and the latter the best product fit:

$$\Delta q > \overline{\theta} - \underline{\theta}. \tag{4}$$

To purchase a product from a firm, a consumer has to visit it which entails a (search) cost. A consumer may visit several firms, one at a time and in a random order so as to extend his choice set of products. Prior to visiting a firm, a consumer is uninformed about its price p_k and its product's characteristics (q_k, θ_{ik}). To capture that some consumers lack the ability to assess products and are susceptible to deception, we assume that there is a fraction ν^N of "naive" consumers. Upon visiting a firm k, a naive consumer only learns the price p_k but neither observes the quality q_k nor the product fit θ_{ik}. Moreover, a naive consumer fails to draw the correct inference between price and quality. More precisely, a naive consumer is an Analogy Based reasoner (Jehiel (2005)) which means that he has correct beliefs about the marginal distribution of qualities and prices in the market, but (possibly incorrectly) believes that they are independent from each other, thus failing to understand that price and quality might be correlated. As a consequence, firms will be able to deceive naive consumers in the sense that their price choice does not betray their quality. In contrast, a fraction ν^S of "sophisticated" consumers, upon visiting a firm, observes prices and product characteristics, and hence recognize deceptive products as what they are.

The sequence of events is as follows. At the outset, firms simultaneously, independently, and once and for all set q_k and p_k. Consumers then search (with perfect recall) until they purchase a product. That is, at any point in time, they decide whether to visit an additional firm at random at search costs $s > 0$, or to buy a product from a previously visited firm (and afterwards leave the market).[15]

Thus, a strategy for a firm is a quality-price combination, and a strategy for consumers is a search rule that specifies whether to end or continue search contingent on the past search history. We adopt a concept of equilibrium which is essentially Perfect Bayesian equilibrium except that naive consumers are Analogy Based reasoners. That is, we define an equilibrium as a strategy profile where firms and all consumers adopt optimal strategies given their beliefs about the other players' strategies. The beliefs of firms and sophisticated consumers are consistent with the other players' strategies, while naive consumers' beliefs are inconsistent in that they wrongly believe that a firm's quality and price choices are independent from each other. In effect, this means that although naive consumers are aware of firms that pursue a deceptive business model, they fail to identify such firms by evaluation of their products and pricing strategy.[16]

In the first step of our analysis, we establish necessary and sufficient conditions for the existence of an equilibrium which displays a segmentation of the market in a "candid" and a "deceptive" segment: a fraction λ of candid firms targets sophisticated consumers by offering quality $\overline{q}$ at a common price $\overline{p}$, and a fraction $1-\lambda$ of deceptive firms targets naive consumers by offering

[15] We thus implicitly assume that consumers have no interest in leaving the market without purchasing a product.

[16] An equilibrium in our model thus formally corresponds to an Analogy Based equilibrium of an associated game in which each firm chooses its quality before its price and the analogy class of an Analogy Based reasoner is the set of all quality choices. In such an equilibrium all firms play the same mixed strategy (we thus do not"purify" the mixed equilibrium strategy) and an Analogy Based reasoner therefore believes that each firm's price and quality choice is independent of each other.

quality $\underline{q}$ at the price $\underline{p}$. Sophisticated consumers search until they find a suitable candid product, whereas naive consumers search until they find a sufficiently cheap (candid or deceptive) product.

Definition 1 *A triple* $(\lambda^*, \overline{p}^*, \underline{p}^*) \in (0,1) \times \mathbb{R}^2$ *is a segmented market equilibrium outcome if there is an equilibrium in which*

- *a fraction* λ^* *of firms offers a candid good with* $q_k = \overline{q}$ *and* $p_k = \overline{p}^*$*;*
- *a fraction* $1-\lambda^*$ *of firms offers a deceptive good with* $q_k = \underline{q}$ *and* $p_k = \underline{p}^*$*;*
- *sophisticated consumers never buy a deceptive good;*
- *naive consumers buy candid and deceptive goods.*

Note that the definition requires the share λ^* of candid firms to be interior. We refer to an equilibrium which supports a segmented market outcome as a segmented market equilibrium.

3 Conditions for market segmentation

In this section, we establish necessary and sufficient conditions for the existence of a segmented market equilibrium. We begin by deriving sophisticated consumers' optimal search rule if they expect a segmented market equilibrium outcome $(\lambda, \overline{p}, \underline{p})$.

Sophisticated consumer search

Because a searching consumer samples the next firm at random from the pool of all firms, a sophisticated consumer's optimal strategy is characterized by a reservation utility $\hat{U}^S$ which is the smallest level of utility that a consumer needs to obtain from purchasing a good so that he stops to search. As is shown by McCall (1970), the reservation utility is the current utility level that leaves a consumer indifferent between ending search in the current period and visiting a single additional firm. In a segmented market $(\lambda, \overline{p}, \underline{p})$, a sophisticated consumer expects to encounter a candid firm with probability λ and a deceptive firm with probability $1-\lambda$. Under the hypothesis that a sophisticated consumer does not buy a deceptive product, the reservation utility is therefore given recursively as

$$\hat{U}^S = \lambda \int_{\underline{\theta}}^{\overline{\theta}} \max\{\overline{q} + \theta - \overline{p}, \hat{U}^S\} \, dF(\theta) + (1-\lambda) \cdot \hat{U}^S - s. \tag{5}$$

A sophisticated consumer does indeed not buy a deceptive product for any match-value if:

$$\hat{U}^S \geq \underline{q} + \overline{\theta} - \underline{p}. \tag{6}$$

It will often be more convenient to work with reservation match-values rather than reservation values. The reservation match-value is defined as the match-value $\hat{\theta} \in \mathbb{R}$ at which a sophisticated consumer is indifferent between buying a candid product with this match-value offered at $\overline{p}$ and continuing search:

$$\overline{q} + \hat{\theta} - \overline{p} \equiv \hat{U}^S, \tag{7}$$

and condition (6), that a sophisticated consumer does not to buy at a deceptive firm, becomes equivalent to

$$\overline{q} + \hat{\theta} - \overline{p} \geq \underline{q} + \overline{\theta} - \underline{p}. \tag{8}$$

To write condition (5) more succinctly, define the function $g : \mathbb{R} \to \mathbb{R}$ by

$$g(z) \equiv \int_{\underline{\theta}}^{\overline{\theta}} \max\{\theta - z, 0\}\, dF(\theta), \tag{9}$$

so that with (7) and (9), (5) can be written as

$$g(\hat{\theta}) = \frac{s}{\lambda}. \tag{10}$$

As we show in Lemma A.1 in the appendix, the function g is strictly decreasing on $(-\infty, \overline{\theta}]$ with $g(-\infty) = \infty$ and $g(\overline{\theta}) = 0$. Therefore, equation (10) has a unique solution.

Naive consumer search

Also the naive consumer's optimal search rule is characterized by a reservation utility. In a segmented market equilibrium, because a naive consumer cannot observe quality and match-values a naive consumer (wrongly) believes that any random next firm supplies him with the average valuation $\mathbb{E}(q + \theta) = \lambda\overline{q} + (1 - \lambda)\underline{q} + 0$, and charges $\overline{p}$ with probability λ and $\underline{p}$ with probability $1 - \lambda$. His reservation utility is therefore given recursively as

$$\hat{U}^N = \lambda \cdot \max\{\hat{U}^N, \mathbb{E}(q) - \overline{p}\} + (1 - \lambda) \cdot \max\{\hat{U}^N, \mathbb{E}(q) - \underline{p}\} - s \tag{11}$$

Because in a segmented market equilibrium a naive consumer purchases candid and deceptive products, it must be optimal for him to buy from any firm, so that

$$\hat{U}^N \leq \mathbb{E}(q) - \overline{p}, \quad \hat{U}^N \leq \mathbb{E}(q) - \underline{p}, \tag{12}$$

and hence, with (11),

$$\hat{U}^{\mathrm{N}} = \lambda \cdot (\overline{q} - \overline{p}) + (1-\lambda) \cdot (\underline{q} - \underline{p}) - s. \tag{13}$$

Therefore, (12) is equivalent to

$$\overline{p} - \underline{p} \leq \frac{s}{1-\lambda} \quad \text{and} \quad \underline{p} - \overline{p} \leq \frac{s}{\lambda}. \tag{14}$$

It will be useful to express a naive consumer's search rule in terms of a reservation price $\hat{p}$ which is the maximal price that he is willing to pay given his beliefs in a segmented market equilibrium, and is implicitly defined as

$$\mathbb{E}(q+\theta) - \hat{p} \equiv \hat{U}^{N}. \tag{15}$$

Thus, by (13),

$$\hat{p} = \lambda \cdot \overline{p} + (1-\lambda) \cdot \underline{p} + s. \tag{16}$$

Effectively, if a firm deviates to a price different from $\underline{p}$ or $\overline{p}$, a naive consumer purchases at the firm whenever the deviation is below $\hat{p}$.[17]

Demand and firm profits

To determine firms' optimal pricing strategies, we now derive demand and profits in the two segments. The following lemma states the profits of a given firm k that charges price p_k taking as given that all other firms and consumers adopt the strategy as specified in a (candidate) equilibrium with outcome $(\lambda, \overline{p}, \underline{p})$. To state the lemma, we define for a given $\overline{p}$ and $\hat{\theta}$:

$$\Pi(p,q,c) = \nu^{N} \cdot (p-c) + \frac{\nu^{S}}{\lambda} \cdot \frac{1 - F\left(\hat{\theta} + (p - \overline{p}) - (q - \overline{q})\right)}{1 - F(\hat{\theta})} \cdot (p-c). \tag{17}$$

Lemma 1 *In a segmented market equilibrium with outcome $(\lambda, \overline{p}, \underline{p})$, the profit of a firm that sets quality q_k and price p_k is given by*

$$\pi(q_k, p_k) = \begin{cases} \Pi(p_k, q_k, c(q_k)) & if \quad p_k \leq \hat{p}; \\ \Pi(p_k, q_k, c(q_k)) - \nu^{N}(p_k - c(q_k)) & if \quad p_k > \hat{p}, \end{cases} \tag{18}$$

[17] Note that although a naive consumer will purchase from the first firm he visits in equilibrium, he will continue search off the path when prices are large. A naive consumer in our setting is therefore different from a Varian (1980) type of "loyal" consumer who always purchases from the first firm he visits no matter what the price. That consumers do not "truly" search and do not visit more than one firm in equilibrium is common if consumers perceive products as homogeneous as, e.g., in Diamond (1971) and Stahl (1989).

where $\hat{p}$ is given by (16). Equilibrium profits are given by

$$\pi(\underline{q},\underline{p}) = \nu^N \cdot (\underline{p} - \underline{c}) \quad \textit{and} \quad \pi(\overline{q},\overline{p}) = (\nu^N + \frac{\nu^S}{\lambda}) \cdot (\overline{p} - \overline{c}). \tag{19}$$

To see what is behind the lemma, note first that the profit of a firm is given as the product of (a) the mass of consumers who visit the firm in its lifetime, (b) the probability that a visitor actually purchases the good, and (c) the firm's mark–up $(p_k - c)$.

In a segmented market equilibrium, (a) comes about as follows. First, recall that all naive consumers buy in the first period and leave the market afterwards. Thus, the mass of naive consumers who visit a given firm in its lifetime is simply equal to ν^N. Second, the mass of sophisticated consumers who visit a given firm in period t is equal to the mass of sophisticated consumers who are still in the market in this period. In a segmented market equilibrium, a sophisticated consumer leaves the market if he is matched with a candid firm that displays a match-value exceeding the consumer's reservation match-value, occurring with probability $\lambda \cdot (1 - F(\hat{\theta}))$. Thus, the mass of sophisticated consumers who visit a given firm in period t is equal to the mass of sophisticated consumers who have not left the market prior to t which is $\nu^S \cdot [1 - \lambda(1 - F(\hat{\theta}))]^t$. Hence, the (expected) mass of sophisticated consumers who visit a given firm in its lifetime is equal to

$$\kappa = \nu^S \sum_{t=0}^{\infty} [1 - \lambda(1 - F(\hat{\theta}))]^t = \frac{\nu^S}{\lambda(1 - F(\hat{\theta}))}. \tag{20}$$

Next, we determine (b), the probability that a visitor actually purchases at the firm. If a sophisticated consumer i visits a firm k with quality q_k and price p_k, he purchases if firm k supplies him with a higher utility than the reservation utility $\hat{U}^S$. By (5) and (7), this is the case if

$$q_k + \theta_{ik} - p_k \geq \overline{q} + \hat{\theta} - \overline{p} \quad \Leftrightarrow \quad \theta_{ik} \geq \hat{\theta} + (p_k - \overline{p}) - (q_k - \overline{q}). \tag{21}$$

Moreover, all naive consumers who visit firm k buy as long as $p_k \leq \hat{p}$. A firm k that sets q_k and $p_k \leq \hat{p}$ therefore earns the profit

$$\pi(q_k, p_k) = \nu^N \cdot (p_k - c(q_k)) + \kappa \cdot \left[1 - F\left(\hat{\theta} + (p_k - \overline{p}) - (q_k - \overline{q})\right)\right] \cdot (p_k - c(q_k)). \tag{22}$$

Inserting (20) delivers (18). Finally, if a firm deviates to a price $p_k > \hat{p}$, it loses the profits $\nu^N(p_k - c(q_k))$ it would otherwise make from naive consumers.

The expressions for equilibrium profits in (19) follow immediately by inserting equilibrium prices $\overline{p}$ and $\underline{p}$ and the respective qualities $\overline{q}$ and $\underline{q}$ in (18), taking into account that $\underline{p} \leq \hat{p}$ in a segmented market equilibrium.

Equilibrium

Our next step is to characterize segmented market equilibrium outcomes formally. We will focus on equilibria with actual consumer search and thus restrict attention to equilibria in which the equilibrium reservation match-value $\hat{\theta}^*$ is in the interior of the support of match-values. From a technical point of view, the restriction to equilibria with interior reservation match-values facilitates tractability, as it allows us to characterize optimal firm behavior by first order conditions.[18] From now on, when we refer to "equilibrium", we mean such an equilibrium in which actual search takes place.

In the next lemma, we provide a system of equations whose solution corresponds to a segmented market equilibrium outcome. To ensure that first order conditions are also sufficient for optimality, we impose the following regularity condition:

$$\frac{f'(\overline{\theta})}{f(\overline{\theta})} \geq -\frac{\nu^S}{\nu^N} \cdot f(\overline{\theta}). \tag{23}$$

Note that the condition always holds with increasing density.[19]

Lemma 2 *Suppose condition (23) holds. Then $(\lambda^*, \overline{p}^*, \underline{p}^*)$ is a segmented market equilibrium out-*

[18]In principle, there may be equilibria with $\hat{\theta}^* = \underline{\theta}$ in which sophisticated consumers buy with probability 1 at the first firm they visit. Intuitively, if sophisticated strictly prefer to purchase at the first firm, then a candid firm's demand would be (locally) inelastic so that it would prefer to raise its price. In these equilibria, however, the fraction of candid firms adjusts such that sophisticated consumers are exactly indifferent between continuing and discontinuing search if they find a product which displays the worst match-value ($\hat{\theta}^* = \underline{\theta}$). A candid firm's profit function then has a kink at the equilibrium price so that its pricing behavior cannot be characterized by first order conditions.

[19]It is standard in the search literature to impose sufficient conditions similar to (23) which ensure that first order conditions are sufficient for profit maximization. In the absence of naive consumers ($\nu^N = 0$), our model is akin to Wolinsky (1986) who only requires that the hazard rate of F be increasing (see the footnote on page 504). This is consistent with (23), because if ν^N is sufficiently small, (23) is satisfied under the mild assumption $f(\overline{\theta}) > 0$, and because our assumption of log-concavity of f implies an increasing hazard rate. A condition similar to (23) is also adopted by Anderson and Renault (1999).

come if and only if there is $\hat{\theta}^* \in (\underline{\theta}, \overline{\theta})$*,* $\hat{p}^* \geq 0$*, and* $\lambda^* \in (0,1)$ *so that:*

$$\begin{aligned} g(\hat{\theta}^*) &= \frac{s}{\lambda^*}, && (24)\\ \hat{p}^* &= \lambda \cdot \overline{p}^* + (1-\lambda) \cdot \underline{p}^* + s, && (25)\\ \overline{p}^* - \overline{c} &= \frac{1 + \lambda^* \frac{\nu^N}{\nu^S}}{h(\hat{\theta}^*)}, && (26)\\ \underline{p}^* &= \hat{p}^* && (27)\\ \left(\nu^N + \frac{\nu^S}{\lambda^*}\right) \cdot (\overline{p}^* - \overline{c}) &= \nu^N \cdot (\underline{p}^* - \underline{c}). && (28) \end{aligned}$$

To understand Lemma 2, notice first that (24) is the same condition as (10), and second that (25) is the same condition as (16). Third, (26) corresponds to the first order condition for $\overline{p}^*$ to be profit maximizing for a candid firm, expressed in terms of the mark–up $\overline{p}^* - \overline{c}$. Because of (23), the first order condition is also sufficient.

Fourth, condition (26) says that $\hat{p}^*$ is the profit maximizing price for a deceptive firm. The reason is that it is optimal for a deceptive firm to target naive consumers and to set prices so as to derive no demand from sophisticates. As a consequence, a deceptive firm's demand is inelastic up to the price $\hat{p}$, and it thus charges $\hat{p}$. In light of (25), this means that

$$\underline{p}^* = \overline{p}^* + \frac{s}{\lambda^*}. \tag{29}$$

Importantly, the mark–up of a deceptive firm is hence bounded from below by Δc, because $\overline{p}^* \geq \overline{c}$.

Fifth, condition (28) says that candid and deceptive firms make the same profits. This has to hold in equilibrium, because otherwise firms in the less profitable segment would want to move to the more profitable one. Thus, the profit that a firm earns by selling deceptive products at a higher mark–up to only naive consumers is equal to the profit that a firm earns by selling candid products at a lower mark–up to naive and sophisticated consumers.

From now on, we impose condition (23) as a general assumption without further mention.

Equilibrium existence and uniqueness

We now ask when an equilibrium exists. Treating the share of candid firms λ as exogenous for the moment, the equilibrium conditions (24) and (26) pin down candid prices as function of λ as

$$\overline{p}^*(\lambda) - \overline{c} \equiv \frac{1 + \lambda \frac{\nu^N}{\nu^S}}{h(g^{-1}(\frac{s}{\lambda}))}, \tag{30}$$

where g^{-1} is the inverse of g as defined in Lemma A.1 in the appendix. Inserting this, together with (29), in firm profits, we obtain the difference between a candid and a deceptive firm's profits as

$$\Delta(\lambda) \equiv \overline{\pi} - \underline{\pi} = \frac{\nu^S}{\lambda} \cdot \left(\overline{p}^*(\lambda) - \overline{c}\right) - \nu^N \cdot \left(\frac{s}{\lambda} + \Delta c\right). \tag{31}$$

The equal profit requirement (28) then amounts to the condition $\Delta(\lambda^*) = 0$. Hence, a segmented market equilibrium exists (and is unique) if the equation $\Delta(\lambda) = 0$ has a (unique) solution $\lambda^* \in (0,1)$ so that $\hat{\theta}^* \in (\underline{\theta}, \overline{\theta})$. As shown in Lemma A.1 in the appendix, $\hat{\theta}^*$ is indeed in $(\underline{\theta}, \overline{\theta})$ if and only if $\lambda^* > -\frac{s}{\underline{\theta}}$. The next proposition summarizes.

Proposition 1 *There is a (unique) segmented market equilibrium outcome* $(\lambda^*, \overline{p}^*, \underline{p}^*)$ *if and only if* $\Delta(\lambda) = 0$ *has a (unique) solution* λ^* *with* $-\frac{s}{\underline{\theta}} < \lambda^* < 1$. *The solution* λ^* *is the equilibrium share of candid firms, and equilibrium prices are pinned down by (24-27).*[20]

The next proposition shows that a unique segmented market equilibrium outcome always exists if search costs are sufficiently small.

Proposition 2 *For sufficiently small search costs, there is a unique segmented market equilibrium outcome.*

The proof shows that the profit difference Δ is positive (resp. negative) if the share of candid firms is small (resp. large). An intermediate value argument then implies that $\Delta(\lambda) = 0$ has a solution in the range $(-\frac{s}{\underline{\theta}}, 1)$. To show uniqueness, it is then sufficient to argue that for sufficiently small search costs, the profit difference is decreasing at any equilibrium share of candid firms:

$$\frac{\partial \Delta}{\partial \lambda}(\lambda^*) < 0. \tag{32}$$

To understand more intuitively the role of search costs for the co-existence of candid and deceptive firms in equilibrium, suppose first that (almost) all firms are candid. Then, as search costs get close to zero, the mark–up for candid firms gets arbitrarily close to zero,[21] whereas the mark–up

[20]Note that a segmented market equilibrium can only exist if $-\frac{s}{\underline{\theta}} < 1$, which is a standard condition (for consumer search to take place) in the search literature (see e.g. Wolinsky (1986) and Anderson and Renault (1999)). We implicitly impose $-\frac{s}{\underline{\theta}} < 1$ below by imposing the stronger condition (65).

[21]Formally, as $s/\lambda \to 0$, equation (24) and the definition of g imply that $\hat{\theta}$ converges to $\overline{\theta}$. It follows that the mark–up $\overline{p} - c$ given by (26) converges to zero because the hazard rate is unbounded.

for deceptive firms is bounded from below by Δc. Therefore, if (almost) all firms are candid, there is a level of search costs below which candid firms earn lower profits than deceptive firms.

On the other hand, if very few firms are candid, the demand for each candid firm is extremely large because the demand from sophisticated consumers is divided up by very few firms. Thus, profits in the candid segment are very large. While the price of deceptive firms, $\underline{p} = \overline{p} + s/\lambda$, is also large if the candid segment is small, this price effect is diminished if search costs s are small, and therefore, if both s and λ are small, candid profits are higher than deceptive profits. As a consequence, there is an intermediate share of candid firms where the profits in the two segments equalize.

In what follows, we assume that search costs are such that a unique segmented market equilibrium exists. This allows us to study the comparative statics of segmented market equilibrium outcomes. To do so, we will abuse notation and denote by $\overline{p}^*, \underline{p}^*$ and $\hat{\theta}^*$ both the equilibrium outcomes which depend only on exogenous parameters, as well as the "best–reply" functions $\overline{p}^*(\cdot), \underline{p}^*(\cdot)$ and $\hat{\theta}^*(\cdot)$ which are pinned down by (24-27) as functions of λ when treated as an independent variable such as in equation (30). In particular, we use total derivatives to indicate changes of equilibrium outcomes, and partial derivatives to indicate changes of the "best–reply", taking λ^* as given.[22]

4 Effects of changes in search costs

In this section, we investigate how changes in the intensity of competition, as captured by changes in search costs, affect market outcomes. We begin by considering how competition affects the size of the two segments and hence the quality provision in the market.

Market segmentation

Our first result says that as search costs vanish, candid firms are entirely driven out of the market.

Proposition 3 *As search costs vanish, candid firms are entirely driven out of the market:* $\lim_{s \to 0} \lambda^* = 0$.

[22] For example, $d\overline{p}^*/ds$ is the change of the candid equilibrium price with respect to s, whereas $\partial \overline{p}^*/\partial s$ is the change of the candid equilibrium price with respect to s when λ^* does not adjust to the change in s. Therefore: $d\overline{p}^*/ds = \partial \overline{p}^*/\partial s + \partial \overline{p}^*/\partial \lambda \cdot d\lambda^*/ds$.

In other words, when only some consumers are vulnerable to deception, then intense competition has the striking consequence that the market will supply only deceptive quality. In a nutshell, the intuition is that vanishing search costs allow sophisticated consumers to compare firms essentially for free. This intensifies competition in the candid segment, thus eliminating candid firms' mark–ups and profits, ultimately leading all firms to adopt the deceptive business model which guarantees positive profits.

This intuition hides the subtlety that what matters for sophisticated consumers is not nominal search costs, but the expected search costs to encounter a candid firm. If nominal search costs are one dollar, and among 100 firms there is only a single candid firm, a sophisticated consumer has to spend on average 100 dollars to find a candid firm. Thus, the *effective search costs* that a sophisticated consumer faces in equilibrium are the expected search costs

$$\sigma^* \equiv \frac{s}{\lambda^*} \tag{33}$$

to find a candid firm. In line with this, observe that candid mark–ups depend on search costs only through effective search costs:

$$\overline{p}^* - \overline{c} = \frac{1 + \lambda^* \frac{\nu^N}{\nu^S}}{h(g^{-1}(\sigma^*))}, \tag{34}$$

and in what follows, we will interpret candid prices as a function of σ rather than s. Therefore, a sophisticated consumer can inspect an additional candid firm essentially for free only if effective search costs vanish.

With this in mind, the intuition behind Proposition 3 can be made more precise. If the share of candid firms λ^* did not converge to zero, then effective search costs σ^* would tend to zero as s approaches zero, and candid firms' mark–ups would erode. Because firms make strictly positive profits in equilibrium, the erosion of mark–ups must be offset by an unbounded increase in the demand of a candid firm which, in turn, requires that the share of candid firms shrinks to zero (recall that in equilibrium a candid firm attracts the demand ν^S/λ^* from sophisticated consumers)—a contradiction.[23]

[23] Our model also displays the property that in the zero search cost limit, the market is frictionless in the sense that effective search costs vanish as search costs get small: $\lim_{s \to 0} \sigma^* = 0$. This also implies that the difference in prices between candid and deceptive firms, which equals σ^* by equation (29), vanishes as search costs go to zero. Formally, we establish that effective search costs vanish as auxiliary result (68) in the proof of Proposition 2.

Proposition 3 shows that very stiff competition where search costs are virtually eliminated is detrimental for the quality provision in the market. We now ask whether it is true in general that lowering search costs, but not all the way to zero, has the detrimental effect that it decreases the share of candid firms in the market. The next result establishes sufficient conditions for this to be the case. Below, we will, however, identify opposite cases where lower search costs increase the number of candid firms. Thus, the effect of search costs on quality provision is, in general, ambiguous. To state the result, we introduce the elasticity of the candid mark–up with respect to effective search costs:[24]

$$\epsilon(\sigma) \equiv \frac{\partial(\overline{p}^* - \overline{c})/\partial\sigma}{(\overline{p}^* - \overline{c})/\sigma} = \frac{h'(\hat{\theta})}{h(\hat{\theta})} \cdot \frac{g(\hat{\theta})}{1 - F(\hat{\theta})}, \quad \text{with } \hat{\theta} = g^{-1}(\sigma). \tag{35}$$

To understand why the elasticity of candid mark–ups depends on the steepness h' of the hazard rate, recall from (34) that $\overline{p}^*$ is inversely related to the hazard rate. This reflects that the hazard rate indicates the percentage increase in the mass of sophisticated consumers who will stop to search and buy the firm's product if it marginally decreases its price. Thus, the larger the hazard rate, the larger is a candid firm's demand elasticity, and the smaller is the equilibrium price. As a consequence, the price a candid firm charges increases sharply in σ if the hazard rate decreases sharply.

Proposition 4 *(i) The share of candid firms increases in s if and only if*

$$\epsilon(\sigma^*) \geq \frac{\sigma^*}{\sigma^* + \Delta c} \tag{36}$$

(ii) The share of candid firms strictly increases in s if $f'(\hat{\theta}^*) \geq 0$.

(iii) The share of candid firms strictly increases in s if search costs are sufficiently small.

Condition (i) says that the share of candid firms increases in search costs if and only if the candid mark–up elasticity is sufficiently large. For example, the condition is met if an increase in effective search costs of one percent results in an increase in a candid firm's mark–up that exceeds one percent. Conditions (ii) and (iii) are sufficient conditions for condition (i) to hold. Notice that (ii) is satisfied for the uniform distribution.[25]

[24]The calculation is provided in Lemma A.3 in the appendix.

[25]While (ii) is stated in terms of the endogenous cutoff $\hat{\theta}^*$, we can turn it into primitive conditions by requiring it to hold for all θ.

Note that, intuitively, the share of candid firms increases in search costs if, all else equal, candid profits increase by more than deceptive profits as search costs increase, because then more firms will adopt the candid business model in response. In order to illuminate the intuition behind Proposition 4, we therefore illustrate the economic forces that drive the evolution of profits in both market segments as search costs change.

Consider an increase of search costs by one dollar. This raises effective search costs by $1/\lambda^*$ dollars, all else equal. Because $\underline{p}^* = \overline{p}^* + \sigma^*$, a deceptive firm's price thus increases by $1/\lambda^*$ dollars more than the price of a candid firm. Now, deceptive and candid firms have the same number ν^N of naive customers, and a candid firm serves an additional ν^S/λ^* sophisticated consumers. As search costs increase marginally, a deceptive firm thus earns $1/\lambda^*$ marginal dollars more than a candid firm per naive customer. Reversely, a candid firm earns $\partial \overline{p}^*/\partial \sigma \cdot 1/\lambda^*$ marginal dollars more from any of its sophisticated consumers. Hence, as search costs increase, the difference between the change in candid and deceptive profits is

$$\frac{\partial \Delta}{\partial s} = \frac{\partial}{\partial s}(\overline{\pi}^* - \underline{\pi}^*) = \frac{\nu^S}{\lambda^*} \cdot \frac{\partial \overline{p}^*}{\partial \sigma} \frac{1}{\lambda^*} - \frac{\nu^N}{\lambda^*}. \tag{37}$$

In what follows, we refer to $\partial \overline{p}^*/\partial \sigma^*$ as the "search effect". The search effect is unambiguously positive:[26] as effective search costs increase, sophisticated consumers become less picky in order to save on search costs, and the critical match-value $\hat{\theta}^*$ goes down. This renders the demand by sophisticated consumers less elastic (as the hazard rate is increasing) which allows firms to increase prices. In economic terms, the last term in (37) captures that (continued) search becomes more costly also for naive consumers, and as a consequence, deceptive firms can enforce even larger mark–ups against them.

The intuition behind Proposition 4 follows now from the observation that (37) is positive if the search effect is strong in the sense that $\partial \overline{p}^*/\partial \sigma$ is large. Then, as search costs rise, candid prices increase so sharply that candid profits grow faster than deceptive profits, and the share of candid firms consequently increases in s. Observe that the size of the search effect $\partial \overline{p}^*/\partial \sigma$ is proportional to the elasticity of candid mark–ups, as

$$\frac{\partial \overline{p}^*}{\partial \sigma} = \epsilon \cdot \frac{\overline{p}^* - \overline{c}}{\sigma^*}. \tag{38}$$

[26]This feature is analogous to the familiar property that in a search model without naive consumers such as Wolinsky (1986) or Anderson and Renault (1999), prices increase in search costs.

Condition (i) is the necessary and sufficient condition for candid mark–ups, in equilibrium, to be sufficiently elastic so that the search effect is sufficiently strong.

The previous considerations suggest that if the candid mark–up elasticity is sufficiently small, then the search effect is so weak that (37) becomes negative. At such a point, the share of candid firms would decrease in s. We now give a parameterized example in order to illustrate that this may indeed be the case. The example takes the exponential distribution, which displays a constant hazard rate, and adapts it to our setting with bounded support. (The calculations behind the example are provided in Appendix B.)

Example *Let* $[\underline{\theta}, \overline{\theta}] = [-1, 1]$*, and define*

$$f^E(\theta) = \begin{cases} e^{-(\theta+1)} & \theta < 0 \\ e^{-1} & \theta \geq 0. \end{cases} \tag{39}$$

Let $f = f^E$*. Then there is an open set of parameters* $s, \overline{q}, \underline{q}, \underline{c}, \overline{c}, \nu^N$ *so that a segmented market equilibrium exists with* $\hat{\theta}^* < 0$ *and*

$$\lambda^* = \frac{\frac{\nu^S}{\nu^N} - s}{\Delta c - 1}. \tag{40}$$

The example has the property that the equilibrium match-value $\hat{\theta}^*$ is below zero, and that, at $\hat{\theta}^*$, the distribution coincides with an exponential distribution. The hazard rate is thus constant and hence the elasticity of candid mark–ups is zero so that the search effect is zero. As a result, condition (i) in Proposition 4 is violated, and the number of candid firms drops as s increases.

Price effects

We now study how equilibrium prices evolve as search costs change. We begin with candid prices. By (34), the effect of a change in search costs on candid prices is given as

$$\frac{d\overline{p}^*}{ds} = \frac{\partial \overline{p}^*}{\partial \sigma}\frac{d\sigma^*}{ds} + \frac{\partial \overline{p}^*}{\partial \lambda}\frac{d\lambda^*}{ds}. \tag{41}$$

To better understand equation (41), we first consider how effective search costs change with search costs.

Lemma 3 *Equilibrium effective search costs are increasing in s:* $d\sigma^*/ds \geq 0$*.*

To see the intuition behind the lemma, suppose to the contrary that effective search costs σ^* were decreasing in s. Hence, the share of candid firms λ^* would increase in s. By (34), this, first of all, would trigger an increase in candid prices, and second, because $\underline{p}^* = \overline{p}^* + \sigma^*$, the difference between deceptive and candid prices would shrink. For firms to nevertheless earn the same profit in the two segments after an increase in s, the demand that each candid firm receives from sophisticated consumers (ν^S/λ^*) would have to drop sharply. As it turns out, such a sharp increase of λ^* is impossible in equilibrium.[27]

Lemma 3 together with the abovementioned fact that the search effect $\partial p^*/\partial\sigma$ is positive, immediately implies that the first term in (41) is positive. The second term in (41) represents a novel effect which is due to the fact the share of candid firms is endogenous in our setting. In what follows, we refer to $\partial p^*/\partial\lambda$ as the "demand composition" effect. Recall that a candid firm's total demand is composed of the less elastic demand of naive consumers and the more elastic demand of sophisticated consumers. If the number of candid firms goes up, sophisticated consumers visit in expectation less firms until they find a suitable candid product to purchase. As a consequence, there are in effect less sophisticated consumers present in the market and the share of naive consumers in a candid firm's demand increases. Because the demand from naive consumers is less elastic than the demand from sophisticated consumers, prices increase. The demand composition effect is thus positive: $\partial\overline{p}^*/\partial\lambda \geq 0$. We conclude that if the share of candid firms increases in s, then the demand composition effect is positive, and we obtain the standard result that equilibrium prices increase in s. Note that because $\underline{p}^* = \overline{p}^* + \sigma^*$, Lemma 3 implies that if candid prices increase in s, so do deceptive prices.

Proposition 5 *If the share of candid firms increases in search costs, that is, $d\lambda^*/ds > 0$, then candid and deceptive prices increase in search costs.*

If on the other hand, the share of candid firms goes down in s, the demand composition effect is negative. We now argue that in this case, candid prices may actually decrease with higher search costs. As with Proposition 4, the size of the search effect depends on the steepness of the hazard rate and is zero in our example with (locally) constant hazard rate. Because in our example, the share of candid firms decreases in s, we thus infer that candid prices decrease in s.

Example (ctd.) *Let $f = f^E$. Then there is an open set of parameters $s, \overline{q}, \underline{q}, \underline{c}, \overline{c}, \nu^N$ so that a*

[27] To the extent that $d\sigma^*/ds > 0$, effective search costs are well-behaved and searching for a suitable candid product becomes more tedious for sophisticated consumers as search friction increase, even if the share of candid firms increases in response.

segmented market equilibrium exists with $\hat{\theta}^* < 0$ and

$$\overline{p}^* - \overline{c} = \frac{\Delta c - s\frac{\nu^N}{\nu^S}}{\Delta c - 1} \tag{42}$$

Thus, candid prices are decreasing in search costs.

Welfare effects

We now turn to the question of whether stiffer competition in form of lower search costs is socially desirable. The welfare of sophisticated and naive consumers is given by (7) and (13), and firm profits are given by (19).[28] Because prices only affect the division but not the size of total welfare, total welfare is thus given by

$$W = \nu^S \cdot [\overline{q} - \overline{c} + \hat{\theta}^*] + \nu^N \cdot [\lambda^* \cdot (\overline{q} - \overline{c}) + (1 - \lambda^*) \cdot (\underline{q} - \underline{c}) - s]. \tag{43}$$

The next proposition shows that the effect of stiffer competition on welfare depends on whether the share of candid firms goes up or down with search costs:

Proposition 6 *(i) If the share of candid firms decreases in search costs, then total welfare decreases in search costs.*

(ii) If the share of candid firms increases in search costs, then total welfare increases in search costs if the efficiency gains from the candid quality, $\Delta q - \Delta c$, are sufficiently large.

In other words, softer competition may be socially desirable if deceptive products are very inefficient. This result is the outcome of three effects which determine the direction of total welfare as search costs increase: First, effective search costs increase (by Lemma 3) so that on average sophisticated consumers end up with a product with a lower match-value ($\hat{\theta}^*$ goes down). Second, the single search that naive consumers conduct gets more costly. These two effects resemble the welfare effects in standard search models and are unambiguously detrimental to total welfare.

In our setting, however, search costs also affect the quality provision in the market. If the share of candid firms goes down with search costs, then naive consumers, on average, consume the inefficient deceptive product more often which reduces total welfare so that part (i) of the Proposition follows immediately. But, if the share of candid firms goes up, as is the case under

[28] Our model displays the property that a naive consumer's belief about his expected welfare is correct, although he wrongly expects to purchase candid products at $\underline{p}^*$ and deceptive products at $\overline{p}^*$. The reason is that he holds correct expectations about the average quality that he purchases and the average price that he pays.

the conditions of Proposition 4, consumption gets more efficient. Thus, if the efficiency gains are sufficiently large, total welfare actually increases with search costs.

We proceed by considering the effects of stiffer competition on consumer welfare and profits separately.

Proposition 7 *If the share of candid firms increases in search costs, then sophisticated consumer welfare decreases, and profits increase in search costs. The effect on (true) naive consumer welfare is not clear–cut.*

Again, the effects depend on how the share of candid firms evolves with search costs. Recall that by Proposition 5 candid and deceptive prices increase in s if the share of candid firms increases in s. In that case, an increase in search costs results in larger effective search costs and prices, and sophisticated consumer welfare therefore unambiguously drops. The profit of a deceptive firm (and hence of any firm) increases, because a deceptive firm's price increases and its demand remains unchanged (ν^N). Finally, naive consumers benefit from the higher likelihood to encounter a candid firm, but suffer from higher market prices, the net effect being ambiguous.[29]

5 Effects of naivete

In this section, we study how changes in the fraction ν^N of naive consumers affect market outcomes. While we have modeled naivety as a feature of consumers, naivety can also be interpreted as a measure of product complexity or the ease with which firms can deceive consumers. In this sense, ν^N may differ across markets.

Market segmentation and prices

Proposition 8 *(i) The share of candid firms decreases in the fraction of naive consumers.*

[29]As we have argued above and illustrated in our example, as search costs increase, candid prices may fall, thus raising the question whether this may lead to increasing sophisticated consumer welfare, or decreasing deceptive prices and profits. As it turns out, this is not the case even in the exponential example where the price decline of candid products induced by increasing search costs is extreme (because the hazard rate is constant). This suggests that in general, when this price decline is more moderate, welfare does not increase and profits do not decrease in search costs. We were, however, not able to verify this.

(ii) Candid and deceptive equilibrium prices increase in the share of naive consumers if the elasticity of the candid mark-up with respect to effective search costs is sufficiently large, that is, if

$$\epsilon(\sigma^*) \geq 1. \tag{44}$$

Part (i) confirms the intuition that as the number of naive consumers grows, the deceptive business model, which targets naive consumers, becomes more attractive. The complete argument, however, is more involved as apart from a direct effect (more consumers are susceptible to deception), an indirect price effect works in the opposite direction. Precisely, as ν^N increases, a candid firm serves relatively more naive consumers whose demand is inelastic and all else equal the firm thus increases its price. This in turn implies that, all else equal, deceptive firms increase their prices by the same amount, because $\underline{p}^* = \overline{p}^* + \sigma^*$. However, as candid firms serve more consumers in equilibrium such an overall increase in prices benefits candid firms more than deceptive ones, and this effect makes offering candid products more attractive. As it turns out, the direct effect dominates the indirect effect, and overall, a firm's incentive to become deceptive indeed increases as ν^N increases.

To understand part (ii), observe that by (34), the overall effect of an increase in the share of naive consumers on candid prices is given by

$$\frac{d\overline{p}^*}{d\,\nu^N} = \frac{\partial \overline{p}^*}{\partial\,\nu^N} + \left(\frac{\partial \overline{p}^*}{\partial \lambda} + \frac{\partial \overline{p}^*}{\partial \sigma}\frac{\partial \sigma^*}{\partial \lambda}\right) \cdot \frac{d\lambda^*}{d\,\nu^N}. \tag{45}$$

The direct effect $\partial \overline{p}^*/\partial\,\nu^N$ is unambiguously positive, because the demand of naive consumers is less elastic than the demand of sophisticated ones. Moreover, $d\lambda^*/D\nu^N > 0$ by part (i). The sign of the term in brackets is, in general, not clear-cut, because as seen above, the search effect $\partial \overline{p}^*/\partial \sigma$ and the demand composition effect $\partial \overline{p}^*/\partial \lambda$ are positive whereas $\partial \sigma^*/\partial \lambda$ is negative (as σ^* decreases in λ). Now, as is argued after Proposition 4, the search effect is large if $\epsilon(\sigma^*)$ is large, and, as is shown in the proof of the Proposition , the bracket in (45) becomes negative if $\epsilon(\sigma^*) \geq 1$. Intuitively, candid prices increase in ν^N if the search effect is sufficiency large, because then the fact that less firms in the market are candid results in larger prices. Finally, observe that deceptive prices increase in ν^N if candid prices do, because $\underline{p}^* = \overline{p}^* + \sigma^*$ and σ^* increases in ν^N, as λ^* decreases in ν^N by (i).

The previous considerations suggest that candid prices may decrease in ν^N if the search effect is sufficiently weak. In our example, where the search effect is zero, this, in fact, turns out to be the case.

Example (ctd.) *Let $f = f^E$. Then there is an open set of parameters $s, \overline{q}, \underline{q}, \underline{c}, \overline{c}, \nu^N$ so that candid prices are given by (42) and decrease in the fraction of naive consumers.*

Welfare effects

Proposition 9 *(i) As the share of naive consumers increases, the welfare of sophisticated and naive consumers decreases and firm profits increase if*

$$\epsilon(\sigma^*) \geq 1. \tag{46}$$

(ii) Total welfare decreases in ν^N if

(a) a candid product is more efficient than a deceptive product irrespective of match values: $\Delta q - \Delta c > \overline{\theta} - \underline{\theta}$, or

(b) if the efficiency gains from purchasing a candid product exceed the expected search costs of finding one: $\Delta q - \Delta c > \sigma^*$.

Part (i) follows straightforwardly from the previous proposition. Because the share of candid firms goes down, sophisticated consumers suffer from larger effective search costs. On top of that, prices increase. Likewise, naive consumers are more likely to end up with a deceptive product and have to pay more (which diminishes both their true and perceived welfare). A deceptive (and hence, a candid) firm's profits increase, because as ν^N increases, it sells deceptive products at a higher price to a larger number of naive consumers.

As to part (ii), each sophisticated and each naive consumer generates less surplus as ν^N increases, because the former buys a less suitable product on average ($\hat{\theta}^*$ goes down) and the latter is more likely to end up with a deceptive product. Welfare thus decreases in ν^N if, in addition, the surplus an additional naive consumer generates is lower than that generated by the sophisticated consumer he replaces. The sufficient conditions stated in part (ii) ensure that this is the case. [30]

6 Policy implications

In this section, we turn to the normative implications of our analysis and discuss policy interventions that may, or may not, improve (consumer) welfare in our framework.

[30]Notice that by a revealed preference argument, a sophisticated consumer obtains, in equilibrium, a larger utility than a naive consumer. This, however, does not imply that a sophisticated consumer generates more additional surplus, because a naive consumer generates a larger profit for firms.

Search costs

Our analysis implies that policy interventions that aim at lowering search costs may backfire. As shown in Proposition 6, the welfare benefits of markets with lower search frictions such as online markets are ambiguous, as they might increase the incentives for firms to become deceptive.

A key measure that aims at lowering search costs in practice is the standardization of price and product information so as to make it easier for consumer to compare prices and products. The harmonization of price information is a common regulatory tool in the EU for financial services (see the discussion in Piccione and Spiegler (2012)) or for grocery products (supermarkets are required to quote the price per 100g).

In practice, search frictions are also reduced by measures that promote e-commerce (such as lowering approval standards to register an online business). Moreover, certain information disclosure duties imposed on firms may reduce search costs if they improve consumers' access to product and price information.[31] Finally, search costs may also be reduced through measures that increase the number of firms in the market, e.g. by reducing the barriers of market entry, because this reduces the time to find (or the distance between) competitors.

Naivete

The welfare implications of policy interventions that aim at reducing the number of naive consumers are described in Proposition 9. Naivete can be reduced by information or education campaigns (e.g. fostering financial literacy in the context of banking services). If naivete is interpreted more broadly as a form of limited attention at the point of purchase, mandatory product labels which categorize products in eye-catching ways ("red", "yellow", "green") and trigger consumer awareness can be seen as a way to reduce naivete.[32]

Price regulation

At a more fundamental level, what ultimately causes welfare losses in our framework is that the presence of naive consumers guarantees positive profits from the deceptive business model which

[31] Information disclosure duties may, however, also make naive consumers aware of the deceptive nature of a product.

[32] Since boundedly rational consumers may have difficulties to process information correctly, such measures may, however, backfire. For example, categories such "green" or "red" may make consumers excessively negligent resp. diligent. Moreover, product labels are also likely to affect search costs. A comprehensive analysis of these issues requires a more detailed model of consumer naivete and is beyond the scope of this paper.

makes price competition dysfunctional. This raises the question whether regulating prices directly may be beneficial in our framework.[33]

To address this question, we consider a situation in which the regulator can impose a price floor $\overline{p}_F$, and we argue that a price floor exceeding the laissez-fare equilibrium price $\overline{p}^*$ can improve total and consumer welfare. To keep the analysis simple, we consider the effect of an exogenously imposed (marginal) increase of the candid equilibrium price from $\overline{p}^*$ to $\overline{p}_F = \overline{p}^* + dp$ for small $dp > 0$. If such an intervention is welfare improving, then it suggests that it is optimal to impose a price floor above the equilibrium price.[34] By the equal profit requirement (31), the share of candid firms that balances the profits in the two segments given an arbitrary candid price $\overline{p}$ is

$$\lambda = \frac{\nu^S \cdot (\overline{p} - \overline{c}) - \nu^N \cdot s}{\nu^N \cdot \Delta c}. \tag{47}$$

The equilibrium effect of a price floor on the share of candid firms is then given by the effect of an increase of the equilibrium price $\overline{p}^*$ to $\overline{p}_F = \overline{p}^* + dp$. A price floor thus improves the quality provision in the market, as λ increases in $\overline{p}$ in (47). Intuitively, a candid firm has a larger demand than a deceptive firm so that as prices increase, the candid business model gets more profitable. This increase in the share of candid firms benefits both sophisticated and naive consumers, as the former face lower effective search costs and the latter are more likely to purchase a candid (more efficient) product. A price floor thus improves total welfare, because paid prices are merely a transfer from the point of view of total welfare. Moreover, if the difference between candid and deceptive quality, Δq, is sufficiently large, then a price floor also improves total consumer welfare. The intuition is that if Δq is sufficiently large, then the gains of naive consumers (who are more likely to purchase a candid product) outweigh the losses (of all consumers) due to higher prices.

Lemma 4 *Imposing a price floor $\overline{p}_F = \overline{p}^* + dp$ always improves total welfare, and improves consumer welfare if the difference in quality between candid and deceptive products, Δq, is sufficiently large.*

Minimum quality standard

[33] We thank Tymon Tatur for raising this question.

[34] It is best to consider our exercise as a first heuristic step of a more rigorous analysis that makes the (realistic) assumption that the regulator lacks relevant knowledge, e.g. about sellers' costs. The regulator then sets a price floor so as to maximize some objective, e.g. expected consumer welfare, anticipating the market outcomes that arises under a price floor regulation. Our analysis for a given cost realization suggests that an optimal price floor will be binding for low cost realizations.

If the regulator can observe quality directly, then imposing a minimum quality standard which requires firms to produce the high quality product and thus effectively bans the deceptive product, would (clearly) improve total and consumer welfare. This is consistent with some regulatory policies that have been adopted in practice.[35]

However, in practice, a minimum quality standard may sometimes be only effective in preventing the most extreme forms of deception, because firms may, in response, design new deceptive products that satisfy the standard but still provide inefficiently low quality and allow firms to charge relatively high mark-ups to naive consumers. In a worst case scenario, a standard only slightly improves the quality of a deceptive product, but does not improve the efficiency of a deceptive product at all. This raises the issue whether, in this case, a minimum quality standard is still welfare improving.

To explore this issue, suppose that a minimum quality standard $q_M \in (\underline{q}, \overline{q})$ with $dq = q_M - \underline{q} > 0$ is in place and firms can respond by offering either the candid product of quality $\overline{q}$ as before or a deceptive product of quality $q_M > \underline{q}$ which is equally inefficient than the "most" deceptive product that has been banned by the standard. That is, production costs for the deceptive quality q_M are $c_M = \underline{c} + (q_M - \underline{q})$. The next lemma shows that in this scenario, imposing a minimum quality standard, which is not too large, still improves total welfare and, depending on the mark-up elasticity with respect to search costs, may improve consumer welfare.

Lemma 5 *For sufficiently small $dq = q_M - \underline{q} > 0$, imposing a minimum quality standard $q_M = \underline{q} + dq$, with $c_M = \underline{c} + dq$, always improves total welfare, and improves consumer welfare if*

$$\epsilon(\sigma^*) \geq 1. \tag{48}$$

The intuition behind the lemma is straightforward. A minimum quality standard implies that a deceptive firm can save dq less on marginal production costs by producing the deceptive quality, but since it is still shunned by sophisticated consumers, it cannot attract additional demand through the higher quality. The deceptive business model thus becomes less attractive and the

[35] For instance, within the EU, consumers are entitled to reimbursement of add-on payments that are not explicitly agreed upon in the initial contract (see Article 22 of the Consumer Rights Directive 2011/83/EU). This can be interpreted as a ban of deception through add-on pricing. Likewise, after the financial crisis, financial supervisory bodies in the EU have been granted the authority to ban "complex" derivatives.

share of candid firms increases as a result of a minimum quality standard. Recall that this improves total welfare, because effective search costs drop and naive consumers are more likely to purchase the candid (more efficient) quality. Moreover, the same reasoning implies that consumer welfare increases if, in addition, candid and deceptive prices fall in response to a minimum quality standard. Now, if candid price fall, then deceptive prices fall, because $\underline{p}^* = \overline{p}^* + s/\lambda^*$ and λ^* increases in response to a minimum quality standard as argued before. Candid prices in turn fall if the increase in the share of candid firms results in lower candid prices, that is, if $d\overline{p}^*/d\lambda < 0$ holds. As is shown in the proof of the lemma, this is the case if $\epsilon(\sigma^*) \geq 1$ holds.

7 Discussion

In the first part of this section, we address the question to what extent our results are robust to variations of our model of consumer naivete. In particular, we discard both properties, one at a time, that distinguish a naive consumer from a (rational) sophisticated consumer in our model, that is, his inability to assess products and his failure to make correct inferences from observed prices.

In the first modification, we model naive consumers as uninformed consumers who lack the ability to assess products but make correct inferences from observed prices. We will argue that in this model variation there does not exist a segmented market equilibrium. The fact that naive consumers fail to make correct inferences is therefore decisive in order to obtain a segmented market equilibrium in which the candid and deceptive business model coexist.

In the second modification, we model naive consumers as consumers who fail to make correct inferences from observed prices but observe a noisy signal about product quality. We show that our central result that candid products are driven out of the market if search frictions vanish still holds if naive consumers observe a sufficiently noisy signal. Therefore, the fact that naive consumer cannot assess products at all should be viewed as a simplifying assumption, whereas the fact that naive consumer struggle with the assessment of products is key.

In the second part of this section, we discuss a model extension where firms can educate naive consumers ("unshrouding"). We show that both deceptive and candid firms have no interest in educating consumers. Hence, even if firms can educate naive consumers, there are no market forces which protect consumers from being deceived.

Beliefs

More specifically, suppose that instead of naive consumers there is a fraction of "uninformed" consumers who could neither observe product quality nor fit but would be otherwise fully rational. Then, upon observing a firm's price, an uninformed consumer would form a belief $\hat{\lambda}(p)$ about the probability that the firm offers high quality. In (a Perfect Bayesian) equilibrium, this belief is consistent with the pricing strategies of firms, but can be arbitrary for prices "off the path" which are not chosen by firms in equilibrium. We impose the restriction that $\hat{\lambda}(p)$ is continuous so that a small change in a product's price does not result in a large change in the consumers' belief.[36] The following lemma shows that there exists no segmented market equilibrium.

Lemma 6 *Suppose there is a fraction of uninformed (but rational) consumers who hold continuous beliefs. Then there is no segmented market equilibrium.*

The intuition behind the lemma is that a separating equilibrium may not exist, because otherwise naive consumers would recognize deceptive products as what they are so that firms would have incentives to offer the efficient quality. On the other hand, a pooling equilibrium may not exist, because otherwise naive consumers would have strict incentives to purchase a deceptive product at the equilibrium price. As a deceptive firm only derives demand from naive consumers in a segmented market equilibrium, it would therefore deviate from the equilibrium price and raise its price.

Noisy signals about product characteristics

First, we consider the case that a naive consumers observes a noisy signal about product quality if he inspects a product but neither observes product fit nor makes correct inferences from observed prices. Precisely, if naive consumer i inspects firm k, then he receives an independent signal $s_{ik} \in \{\underline{s}, \overline{s}\}$ which indicates whether the product is more or less likely to exhibit high quality. Precisely, let $\text{Prob}(\overline{s}|\overline{q}) = \text{Prob}(\underline{s}|\underline{q}) = 1/2 + \varepsilon$ so that $\varepsilon \in [0, 1/2]$ indicates how noisy the signal is. The following lemma shows that if ε sufficiently small, then our central insight from Proposition 3 carries over, that is, as search costs vanish, candid firms are driven out of the market.

[36] Absent any belief restriction, there exists a large number of pooling equilibria which rely on the fact that consumers believe that a firm offers low quality whenever it deviates from the equilibrium price.

Lemma 7 *Suppose that naive consumers observe a noisy signal about product quality. Then, if the signal is sufficiently noisy (ε sufficiently small), candid firms are entirely driven out of the market as search costs vanish:* $\lim_{s \to 0} \lambda^* = 0$.

Intuitively, if the signal is too informative and (almost) fully reveals the product's quality to a naive consumer, then there does not exist an equilibrium in which firms offer the deceptive product. The reason is that otherwise there are efficiency gains from trading the efficient, candid quality left on the table, which could split between the firm and its customers, because all consumers recognize products as what they are.[37] If the signal about product quality is, however, sufficiently noisy, then some firms offer the deceptive quality but lower their prices slightly (in comparison to the market outcome in our basic model) such that still all naive consumer, including those that receive a bad signal about the product's quality, purchase its product. This slightly reduces prices and profits in the deceptive segment, yet the equilibrium remains to be a segmented market equilibrium. Moreover, deceptive firms sell small volumes at a large mark–up whereas candid firms sell large volumes at small mark–ups. Loosely speaking, this implies that the intuition of Proposition 3 carries over: as search costs vanish, mark–ups in the candid segment erode and must be off-set by an unbounded increase in demand for candid firms, because the profits in the deceptive segment are bounded away from zero.

In a similar way, it can be shown that our main result carries over if naive consumers receive a sufficiently noisy signal about product fit. Similarly, however, our equilibrium breaks down if the signal is too informative which for example would be the case if naive consumers could perfectly observe product fit.

Shrouding

The literature on consumer exploitation discusses the question whether firms have incentives to educate naive consumers by "unshrouding" the deceptive features of the low quality products in the market (see Gabaix and Laibson (2006), Heidhues et al. (2017)). This is an important question because it bears on whether and how these markets should be regulated. To shed light on this question in our framework, suppose that any firm had the opportunity to "unshroud" in

[37] Precisely, the firm could offer the candid instead of the deceptive quality and raise its mark–up by an amount equal to the efficiency gains. As a naive consumers recognizes products as what they are, the firm still offers the same utility to the consumer (and hence looses no demand) but obtains a larger mark–up.

the sense that after its quality choice it can credibly disclose its product features q and θ, and in effect "turn" any visiting consumer into a sophisticated one. We may think of two scenarios here. In the first scenario, the firm can only "unshroud" yet not adjust its price. While it is obvious that no deceptive firm has an interest to "unshroud", this is also true for any candid firm, since the demand from naive consumers is inelastic and these consumers, in contrast to sophisticated ones, purchase the firm's product in any case at the equilibrium price. In that sense a "business stealing effect" that could render "unshrouding" profitable is absent in our model, as "unshrouding" does not generate any additional demand for candid firms from otherwise naive consumers. Note that this logic carries over to the second scenario in which the firm can "unshroud" and charge a price different from the equilibrium price, because, intuitively, if all consumers were sophisticated, a candid firm would optimally lower its price, thus making smaller profits. We conclude that even if firms could unshroud their products, market forces do not protect consumers from being deceived.

8 Conclusion

In this paper, we integrate consumer naivete in a search market model. The key idea is that the presence of naive consumers prevents competitive forces to unfold, because dumping low quality products on naive consumers is a safe profit haven. In fact, it is precisely in seemingly competitive markets with low search frictions where sellers have strong incentives to adopt the deceptive business model. Policy measures that aim at lowering search frictions should therefore be considered with caution.

A Appendix

Lemma A.1 *(i) g is strictly decreasing on $(-\infty, \overline{\theta}]$ with $g(-\infty) = \infty$, $g(\underline{\theta}) = -\underline{\theta}$, and $g(\overline{\theta}) = 0$, and $g(z) = 0$ for all $z \geq \overline{\theta}$. Moreover, $g' = -(1-F)$.*
(ii) The inverse of g, denoted by g^{-1}, is well–defined and strictly decreasing on the domain $(0, \infty)$. By convention, we define $g^{-1}(0) = \overline{\theta}$. Note also that $g^{-1}(-\underline{\theta}) = \underline{\theta}$.

Proof of Lemma A.1 To see (i), note that for all $z \leq \overline{\theta}$, we have $g'(z) = -(1 - F(z))$, so that g is strictly decreasing on $(-\infty, \overline{\theta})$. The values of g stated in the lemma follow straightforwardly by plugging in the respective arguments. Part (ii) is an immediate consequence of (i). ■

Proof of Lemma 1 The proof follows from the main text. ■

Lemma A.2 *Let*

$$\overline{p} = \frac{1 + \lambda \frac{\nu^N}{\nu^S}}{h(\hat{\theta})} + c, \quad p_L = (\underline{\theta} - \hat{\theta}) + \overline{p}, \quad p_H = (\overline{\theta} - \hat{\theta}) + \overline{p}. \tag{49}$$

Then the function $\Pi(\cdot, \overline{q}, c)$ as defined in (17) is maximized on the domain (p_L, p_H) by $p^* = \overline{p}$.

Proof of Lemma A.2 Notice first that $p^* = \overline{p}$ is indeed in (p_L, p_H). The necessary first order condition for $p^* = \overline{p}$ to maximize $\Pi(\cdot, \overline{q}, c)$ is

$$0 = \frac{\partial \Pi(p^*, \overline{q}, c)}{\partial p} = \nu^N + \frac{\nu^S}{\lambda} - \frac{\nu^S}{\lambda} \cdot \frac{f\left(\hat{\theta}\right)}{1 - F(\hat{\theta})} \cdot (p^* - \overline{c}). \tag{50}$$

By (49), $p^* = \overline{p}$ therefore satisfies the first order condition.

To see that $p^* = \overline{p}$ is a global maximizer, we show that $\Pi(\cdot, \overline{q}, c)$ is quasi-concave on (p_L, p_H). In fact, we show the stronger property that $\Pi(\cdot, \overline{q}, c)$ is log-concave. To see this, let $\tau = \tau(p) = \hat{\theta} + (p - \overline{p})$ and define

$$D(p) \equiv \nu^N + \frac{\nu^S}{\lambda} \cdot \frac{1 - F(\tau)}{1 - F(\hat{\theta})}. \tag{51}$$

Observe that $\Pi(p, \overline{q}, c) = D(p) \cdot (p - c)$, and recall that the product of log-concave functions is log-concave. Because $(p - c)$ is log-concave in p, $\Pi(\cdot, \overline{q}, c)$ is therefore log-concave if D is log-concave. To show this, assume to the contrary that there is a p so that $(\log D)''(p) > 0$. As we show below,

$$(\log D)''(p) > 0 \quad \Rightarrow \quad (\log D)''(p_H) > 0. \tag{52}$$

The right inequality in turn is equivalent to[38]

$$D''(p_H)\cdot D(p_H) - D'(p_H)^2 > 0. \tag{53}$$

With $\alpha \equiv \frac{\nu^S}{\lambda(1-F(\hat{\theta}))}$, we have

$$D'(p) = -\alpha f(\tau) \quad \text{and} \quad D''(p) = -\alpha f'(\tau). \tag{54}$$

Because $D' < 0$ and $D > 0$, dividing (53) by $D'(p_H)\cdot D(p_H)$ and re-arranging terms yields

$$\frac{f'(\overline{\theta})}{f(\overline{\theta})} < -\frac{\alpha f(\overline{\theta})}{\nu^N + \alpha(1-F(\overline{\theta}))} = -\frac{\alpha f(\overline{\theta})}{\nu^N} < -\frac{\nu^S}{\nu^N} f(\overline{\theta}), \tag{55}$$

where the final inequality follows from $\alpha > \nu^S$. But this inequality contradicts (23) which establishes that D is quasi-concave, and hence $\Pi(\cdot, \overline{q}, c)$ is log-concave as desired.

To complete the proof, we have to show (52). Notice that with (54) we have

$$(\log D)''(p) = \frac{D'(p)}{D(p)} \cdot \left[\frac{f'(\tau)}{f(\tau)} - \frac{D'(p)}{D(p)}\right], \tag{56}$$

and that $\frac{D'(p)}{D(p)}$ is negative by (54). Therefore, if the term in the square bracket is decreasing at p whenever $(\log D)''(p)$ is positive, then $(\log D)''(p)$ remains positive once it is positive and (52) follows. To see that the term in the square brackets is indeed decreasing at p, note that $(\log D)''(p) > 0$ implies that $(\log D)'(p) = \frac{D'(p)}{D(p)}$ is increasing at p and our assumption of f log-concave implies that $\frac{f'(\tau)}{f(\tau)}$ is decreasing at p. ■

Proof of Lemma 2 "⇐": Let $\theta^* \in (\underline{\theta}, \overline{\theta})$ and $\lambda^* \in (0,1)$ be given and suppose (24–28) hold. We first show that consumer behavior is as required in a segmented market equilibrium. By (29), we have $\underline{p}^* = \overline{p}^* + s/\lambda^*$. Condition (14) is thus met. Therefore, naive consumers purchase both candid and deceptive products, and (25) re-states their optimal search rule (16). Also condition (8) is met, because $\underline{p}^* > \overline{p}^*$ by (29) and $\overline{\theta} - \hat{\theta} < \Delta q$ where the latter inequality follows from $\theta^* \in (\underline{\theta}, \overline{\theta})$ and assumption (3). Therefore, sophisticated consumer indeed prefer not to purchase deceptive products, and (24) simply re-states their optimal search rule (24).

Consequently, because consumer behavior conforms with a segmented market equilibrium, a firm's profit function is given by Lemma 1. What remains to be shown is that $(\overline{q}, \overline{p}^*)$ and $(\underline{q}, \underline{p}^*)$ with $\underline{p}^*$ and $\overline{p}^*$ as defined in (26) and (29) are indeed profit maximizing strategies for firms.

[38]Note that $\frac{d^2}{dp_k^2}\log D = \frac{d}{dp_k}(D'/D) = 1/D^2 \cdot (D''D - D'^2)$.

Notice that by (28) firms earn equal profits in the two segments so that it suffices to show that $p_k = \overline{p}^*$ is profit maximizing given $\overline{q}$, and $p_k = \underline{p}^*$ is profit maximizing given $\underline{q}$, as in this case, (28) ensures that a deviation to the other segment is not profitable.

We begin with a candid firm with $q_k = \overline{q}$. By Lemma 1, we have that

$$\pi(\overline{q}, p_k) \leq \Pi(p_k, \overline{q}, \overline{c}) \quad \text{for all } p_k, \quad \text{and} \quad \pi(\overline{q}, \overline{p}^*) = \Pi(\overline{p}^*, \overline{q}, \overline{c}), \tag{57}$$

Hence, Lemma A.2 implies that $\overline{p}^*$ is profit maximizing within the price range $[p_L, p_H]$. Hence, it remains to show that a candid firm cannot improve by choosing a price outside of $[p_L, p_H]$.

Consider first a price $p_k < p_L$. At such a price, any sophisticated and naive consumer for any match-value realization strictly prefers to purchase at the current firm k rather than visit the next firm. Thus, the firm's demand would be locally inelastic, and it would be profitable for the firm to slightly increase the price. Hence, $p_k < p_L$ is not optimal.

Consider next a price $p_k > p_H$. At such a price any sophisticated consumer for any match-value realization strictly prefers to visit the next firm rather than purchase at the current firm k, and the firm at most sells to naive consumers. So suppose that a naive consumer indeed purchases a candid product (and hence also a deceptive product) at p_k such that the firm makes profits of $\nu^N \cdot (p_k - \overline{c})$. We infer:

$$\pi(\overline{q}, p_k) = \nu^N \cdot (p_k - \overline{c}) < \nu^N \cdot (p_k - \underline{c}) = \pi(\underline{q}, p_k) \leq \pi(\underline{q}, \underline{p}^*) = \pi(\overline{q}, \overline{p}^*), \tag{58}$$

where the second inequality follows from $\overline{c} > \underline{c}$ and the third equality follows, because at p_k a deceptive firm only derives demand from naive consumers, as a sophisticated consumer who does not purchase a candid product at p_k neither purchases a deceptive product at p_k. The fourth inequality follows, as $\underline{p}^*$ is optimal given $\underline{q}$ (which we show next), and the final equality follows from (28). This proves that it is optimal for a candid firm to charge $\overline{p}^*$.

To see that it is optimal for a deceptive firm to charge $\underline{p}^*$, consider first a price $p_k > \underline{p}^*$. Because condition (8) is met in (the candid) equilibrium, at $\underline{p}^*$ (and any larger price $p_k > \overline{p}^*$) a deceptive firm derives no demand from sophisticated consumers. Moreover, because $\underline{p}^* = \hat{p}$ by (25), we have $p_k > \hat{p}$, so that the firm neither derives demand from naive consumers. Consequently, a deviation to $p_k > \underline{p}^*$ is not profitable.

Next, consider $p_k < \underline{p}^*$. Observe that because the demand of naive consumers is inelastic up to the price $\hat{p}$ and $\underline{p}^* = \hat{p}$ by (25), setting a price $p_k < \underline{p}^*$ can only be profitable if it generates additional demand from sophisticated consumers, that is, if $p_k < p_H - \Delta q$.[39] Now, at very low prices $p_k < p_L - \Delta q$, any visiting naive and sophisticated consumer would purchase the firm's product so that the firm's demand would be perfectly inelastic. Accordingly, it would be profitable to raise prices. Hence, $p_k < p_L - \Delta q$ is not optimal.

Thus, it remains to consider prices $p_k \in [p_L - \Delta q, p_H - \Delta q]$. Observe that as $p_k < \hat{p}$, it holds that

$$\pi(\underline{q}, p_k) = \Pi(p_k, \underline{q}, \underline{c}) \leq \Pi(p_k, \underline{q}, \overline{c} - \Delta q), \tag{59}$$

where the second inequality follows, because $\Pi(p, q, c)$ is decreasing in c and because $\Delta q > \Delta c$ by (3). From definition (17), we infer that

$$\Pi(p_k, \overline{q}, \overline{c} - \Delta q) = \Pi(p_k + \Delta q, \overline{q}, \overline{c}), \tag{60}$$

and note that $p_k + \Delta q \in [p_L, p_H]$, because $p_k \in [p_L - \Delta q, p_H - \Delta q]$. By Lemma A.2, we thus have

$$\Pi(p_k + \Delta q, \overline{q}, \overline{c}) \leq \Pi(\overline{p}^*, \overline{q}, \overline{c}). \tag{61}$$

With (57) and (28), we can conclude that

$$\pi(\underline{q}, p_k) \leq \Pi(\overline{p}^*, \overline{q}, \overline{c}) = \pi(\overline{q}, \overline{p}^*) = \pi(\underline{q}, \underline{p}^*). \tag{62}$$

Hence, a deviation from $\underline{p}^*$ to $p_k \in [p_L - \Delta q, p_H - \Delta q]$ is not profitable for a deceptive firm. This completes the proof of the "$\Leftarrow$"- part.

"$\Rightarrow$ ": If there is a segmented market equilibrium, then there must exist $\hat{\theta}^* \in (\underline{\theta}, \overline{\theta})$ and $\lambda^* \in (0, 1)$ by definition and, a sophisticated consumer's optimal search rule satisfies (10) which is the same condition as (24). A naive consumer's optimal search rule in turn satisfies (16) which is the same condition as (25). In a segmented market equilibrium, a deceptive firm only derives inelastic demand from naive consumers so that it charges the largest price such that naive consumers yet purchase its product. We hence have $\underline{p}^* = \hat{p}$ which is the same condition as (27). From (25) and (27) we conclude that $\overline{p}^* < \hat{p}$ in any segmented market equilibrium. Because Lemma 1 applies, a candid firm's profits are thus given by $\pi(\overline{q}, p) = \Pi(p, \overline{q}, \overline{c})$ for p from an open interval around $\overline{p}^*$, (as $\hat{\theta}^* \in (\underline{\theta}, \overline{\theta})$ and $\overline{p}^* < \hat{p}$). Hence, $\pi(\overline{q}, p)$ is differentiable at $\overline{p}^*$ and the corresponding first

[39] By definition, p_H is the largest price that a sophisticated is willed to pay for a candid product that displays the best match-value $\overline{\theta}$. Therefore, at $p_k \geq p_H - \Delta q$ a deceptive firm derives no demand from sophisticated consumers.

order condition for $\overline{p}^*$ to be profit maximizing, stated in (50), implies (26). Finally, candid and deceptive firms must make equal profits, as otherwise, firms in the less profitable segment would move to the other segment, and (28) follows from (19) in Lemma 1.

Proof of Proposition 1 By Lemma 2, it is sufficient to show that (24-28) has a solution $(\hat{\theta}^*, \lambda^*, \hat{p}^*, \overline{p}^*, \underline{p}^*)$ with $\hat{\theta}^* \in (\underline{\theta}, \overline{\theta})$, $\hat{p}^* \geq 0$ and $\lambda^* \in (0,1)$ if and only if $\Delta(\lambda^*) = 0$ and $\lambda^* > -\frac{s}{\underline{\theta}}$.

Indeed, it follows by Lemma A.1 that $\hat{\theta}^* = g^{-1}(\frac{s}{\lambda})$ is a solution to (24) with $\hat{\theta}^* \in (\underline{\theta}, \overline{\theta})$ if and only if $\lambda^* > -\frac{s}{\underline{\theta}}$. Note further that with (24), (26) is equivalent to

$$\overline{p}^* - \overline{c} = \frac{1 + \lambda \frac{\nu^N}{\nu^S}}{h(g^{-1}(\frac{s}{\lambda^*}))}. \tag{63}$$

Inserting this and (29) into (28) and re–arranging terms yields that λ^* is a solution to

$$\nu^N \cdot (\frac{s}{\lambda} + \Delta c) = \frac{\nu^S}{\lambda} \cdot \frac{1 + \lambda \frac{\nu^N}{\nu^S}}{h(g^{-1}(\frac{s}{\lambda}))} \tag{64}$$

which, after subtracting the right from the left hand side, is equivalent to $\Delta(\lambda^*) = 0$, and this completes the proof. ■

Proof of Proposition 2 *Existence* An intermediate value argument implies that $\Delta(\lambda) = 0$ has a solution in the range $(-\frac{s}{\underline{\theta}}, 1)$ if

$$\Delta\left(-\frac{s}{\underline{\theta}}\right) > 0 \quad \text{and} \quad \Delta(1) < 0. \tag{65}$$

We show that (65) is satisfied if s is sufficiently small.

To see that the left inequality in (65) is met for small s, insert $\lambda = -\frac{s}{\underline{\theta}}$ into Δ to obtain

$$\Delta\left(-\frac{s}{\underline{\theta}}\right) = \left(-\frac{\nu^S \underline{\theta}}{s} + \nu^N\right) \cdot \frac{1}{h(g^{-1}(-\underline{\theta}))} - \nu^N(-\underline{\theta} + \Delta c). \tag{66}$$

Because $\underline{\theta} < 0$, this expression converges to $+\infty$ as $s \to 0$, as desired.

To see that the right inequality in (65) is met for small s, insert $\lambda = 1$ into Δ to obtain

$$\Delta(1) = \frac{1}{h(g^{-1}(s))} - \nu^N(s + \Delta c). \tag{67}$$

Recall from (9) that $g(\overline{\theta}) = 0$. Therefore, $\lim_{s \to 0} g^{-1}(s) = \overline{\theta}$, and hence, because $\lim_{\theta \to \overline{\theta}} h(\theta) = \infty$ by assumption, we obtain that $\lim_{s \to 0} \Delta(1) = -\nu^N \Delta c < 0$, and this establishes (65).

Uniqueness To show that $\Delta(\lambda) = 0$ has a unique solution for small s, we show first the auxiliary claim

$$\lim_{s\to 0} \frac{s}{\lambda^*(s)} = 0 \quad \text{for all } \lambda^* = \lambda^*(s) \text{ with } \Delta(\lambda^*) = 0. \tag{68}$$

It is clearly sufficient to show this for $\underline{\lambda}^* = \min\{\lambda^* | \Delta(\lambda^*) = 0\}$ (which exists by the continuity of Δ.) Indeed, suppose to the contrary that that there is a (sub)sequence s_n, $n = 1, 2, \ldots$ with $s_n \to 0$ so that the sequence $s_n/\underline{\lambda}^*(s_n)$ is bounded from below by some $\zeta > 0$, that is, $s_n/\underline{\lambda}^*(s_n) > \zeta$ for all n. This first of all implies that $\lim_{n\to\infty} \underline{\lambda}^*(s_n) = 0$. Moreover, it implies that there is some $\theta_0 < \overline{\theta}$ so that $g^{-1}(\frac{s_n}{\underline{\lambda}^*(s_n)}) < \theta_0$ for all n by Lemma A.1. Together with (31), it thus follows that

$$\lim_{n\to\infty} \Delta(\lambda^*(s_n), s_n) \geq \lim_{n\to\infty} \left\{ \left(\frac{\nu^S}{\lambda^*(s_n)} + \nu^N \right) \cdot \frac{1}{h(\theta_0)} - \nu^N \left(\frac{s_n}{\lambda^*(s_n)} + \Delta c \right) \right\} = \infty, \tag{69}$$

a contradiction to assumption that $\Delta(\lambda^*(s_n), s_n) = 0$ for all n, and this establishes (68).

To complete the proof of uniqueness, we show

$$\frac{\partial \Delta}{\partial \lambda}(\lambda^*) < 0 \quad \text{for all } \lambda^* \text{ with } \Delta(\lambda^*) = 0 \text{ and } s \text{ sufficiently small.} \tag{70}$$

This is sufficient for uniqueness by (65). Indeed, to compute $\partial\Delta/\partial\lambda$, observe first that by (26)

$$\begin{aligned}
\frac{\partial}{\partial \lambda}(\overline{p}^* - \overline{c}) &= \frac{\nu^N}{\nu^S} \cdot \frac{1}{h(\hat{\theta}^*)} - \left(1 + \lambda^* \frac{\nu^N}{\nu^S}\right) \cdot \frac{h'(\hat{\theta}^*)}{h(\hat{\theta}^*)^2} \cdot \frac{\partial \hat{\theta}^*}{\partial \lambda} && (71)\\
&= \frac{\nu^N}{\nu^S} \cdot \frac{1}{h(\hat{\theta}^*)} - \left(1 + \lambda^* \frac{\nu^N}{\nu^S}\right) \cdot \frac{h'(\hat{\theta}^*)}{h(\hat{\theta}^*)^2} \cdot \frac{1}{1 - F(\hat{\theta}^*)} \cdot \frac{s}{\lambda^{*2}}, && (72)
\end{aligned}$$

where in the second line, we have used first that the equilibrium property $g(\hat{\theta}^*) = s/\lambda^*$ implies $\partial\hat{\theta}^*/\partial\lambda^* = -1/g'(\hat{\theta}^*) \cdot (s/\lambda^{*2})$, and second that $g'(\hat{\theta}^*) = -(1 - F(\hat{\theta}^*))$ by Lemma A.1.

Because the hazard rate is increasing, we therefore obtain that $\frac{\partial}{\partial\lambda}(\overline{p}^* - \overline{c}) \leq \frac{\nu^N}{\nu^S} \cdot \frac{1}{h(\hat{\theta}^*)}$, and hence

$$\begin{aligned}
\frac{\partial \Delta}{\partial \lambda}(\lambda^*) &= \left(-\frac{\nu^S}{\lambda^{*2}}\right) \cdot (\overline{p}^* - \overline{c}) + \frac{\nu^S}{\lambda^*} \cdot \frac{\partial}{\partial \lambda}(\overline{p}^* - \overline{c}) + \nu^N \cdot \frac{s}{\lambda^{*2}} && (73)\\
&\leq \left(-\frac{\nu^S}{(\lambda^*)^2}\right) \cdot (\overline{p}^* - \overline{c}) + \frac{\nu^N}{\lambda^*} \cdot \frac{1}{h(\hat{\theta}^*)} + \nu^N \cdot \frac{s}{(\lambda^*)^2} && (74)\\
&= -\frac{1}{\lambda^*}\Delta(\lambda^*) - \frac{1}{\lambda^*}\Delta c + \frac{\nu^N}{\lambda^*} \cdot \frac{1}{h(\hat{\theta}^*)} && (75)\\
&= -\frac{1}{\lambda^*}\Delta c + \frac{\nu^N}{\lambda^*} \cdot \frac{1}{h(\hat{\theta}^*)}, && (76)
\end{aligned}$$

where in the last line, we have used that $\Delta(\lambda^*) = 0$. Now observe that (68) implies that $\hat{\theta}^* = g^{-1}(s/\lambda^*) \to \overline{\theta}$ by Lemma A.1. Since the hazard rate is unbounded, expression (76) becomes negative for sufficiently small s. This establishes (70) and completes the proof. ■

Lemma A.3 *We have*

$$\frac{\partial(\overline{p}^*(\sigma)-\overline{c})/\partial\sigma}{(\overline{p}^*(\sigma)-\overline{c})/\sigma} = \frac{h'(\hat{\theta})}{h(\hat{\theta})} \cdot \frac{g(\hat{\theta})}{1-F(\hat{\theta})}, \quad \textit{with } \hat{\theta} = g^{-1}(\sigma). \tag{77}$$

Proof of Lemma A.3 Differentiating $\overline{p}^*$ as given in (34) with respect to σ and inserting σ^* yields

$$\frac{\frac{\partial}{\partial\sigma}(\overline{p}^* - \overline{c})}{\overline{p}^* - \overline{c}} = -\frac{h'(g^{-1}(\sigma^*))}{h(g^{-1}(\sigma^*))} \cdot \frac{\partial g^{-1}(\sigma^*)}{\partial\sigma}. \tag{78}$$

Observe that $\partial g^{-1}(\sigma^*)/\partial\sigma = 1/g'(g^{-1}(\sigma^*))$, and from Lemma A.1, that $g' = -(1-F)$. We thus obtain:

$$\frac{\frac{\partial}{\partial\sigma}(\overline{p}^* - \overline{c})}{\overline{p}^* - \overline{c}} \cdot \sigma^* = \frac{h'(g^{-1}(\sigma^*))}{h(g^{-1}(\sigma^*))} \cdot \frac{\sigma^*}{1-F(g^{-1}(\sigma^*))} \tag{79}$$

The desired result follows then from $\hat{\theta}^* = g^{-1}(\sigma^*)$ by (24). ■

Proof of Proposition 3 Contrary to the claim suppose that there is a (sub)sequence s_n, $n = 1, 2, \ldots$ with $s_n \to 0$ so that $\lambda^*(s_n)$ is bounded from below by some $\underline{\lambda}$, that is, $\lambda^*(s_n) > \underline{\lambda}$ for all n. Then, $\lim_{n\to\infty} s_n/\lambda^*(s_n) = 0$ holds, and Lemma A.1 implies that $\lim_{n\to\infty} g^{-1}(\frac{s_n}{\lambda^*(s_n)}) = \overline{\theta}$. We can then infer that

$$\lim_{n\to\infty} \Delta(\lambda^*(s_n), s_n) < \left(\frac{\nu^S}{\underline{\lambda}} + \nu^N\right) \cdot \lim_{\theta\to\overline{\theta}} \frac{1}{h(\theta)} - \nu^N \Delta c < 0, \tag{80}$$

where the last inequality follows from the fact that the hazard rate diverges as θ approaches $\overline{\theta}$. This contradicts the equilibrium condition that $\Delta(\lambda^*(s_n), s_n) = 0$ for all n. ■

Proof of Proposition 4 We have to show that the stated conditions imply that $d\lambda^*/ds > 0$. By the equilibrium property $\Delta(\lambda^*, s) = 0$, we have that $d\lambda^*/ds = -(\partial\Delta/\partial s)/(\partial\Delta/\partial\lambda)$. Because $\partial\Delta(\lambda^*)/\partial\lambda < 0$ by (70), it suffices to show that $\partial\Delta/\partial s > 0$.

As is argued in the text, differentiating (31) with respect to s yields (37):

$$\frac{\partial\Delta}{\partial s} = \frac{\nu^S}{\lambda^*} \cdot \frac{\partial\overline{p}^*}{\partial\sigma} \frac{1}{\lambda^*} - \frac{\nu^N}{\lambda^*}, \tag{81}$$

which by the definition of ϵ in (35) is equivalent to

$$\frac{\partial\Delta}{\partial s} = \frac{\nu^S}{\lambda^*} \cdot (\overline{p}^* - \overline{c}) \cdot \epsilon(\sigma^*) \cdot \frac{1}{s} - \frac{\nu^N}{\lambda^*}. \tag{82}$$

We now prove parts (i) to (iii), beginning with part (i). By (31), $\Delta(\lambda^*) = 0$ is equivalent to

$$\frac{\nu^S}{\lambda^*} \cdot (\overline{p}^* - \overline{c}) = \nu^N \cdot \left(\frac{s}{\lambda^*} + \Delta c \right), \tag{83}$$

and (82) becomes

$$\frac{\partial \Delta}{\partial s} \;\; = \;\; \nu^N \cdot \left(\frac{s}{\lambda^*} + \Delta c \right) \cdot \epsilon(\sigma^*) \cdot \frac{1}{s} - \frac{\nu^N}{\lambda^*} = \frac{\nu^N}{\lambda^*} \cdot \left[\left(\frac{s + \lambda^* \Delta c}{s} \right) \cdot \epsilon(\sigma^*) - 1 \right]. \tag{84}$$

Thus, condition (i) is necessary and sufficient for $\partial \Delta / \partial s > 0$.

We next show part (ii). Notice first that a straightforward calculation yields that

$$\frac{h'(\hat{\theta}^*)}{h(\hat{\theta}^*)} = \frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*)} + h(\hat{\theta}^*) \tag{85}$$

and $\epsilon(\sigma^*)$ in (35) becomes

$$\epsilon(\sigma^*) = \left(\frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*)} + h(\hat{\theta}^*) \right) \cdot \frac{g(\hat{\theta}^*)}{1 - F(\hat{\theta}^*)} \tag{86}$$

with $\hat{\theta} = g^{-1}(\sigma)$. Inserting this and $\overline{p}^*$ from (34) in (82) yields

$$\frac{\partial \Delta}{\partial s} = \left(\frac{\nu^S}{\lambda^*} + \nu^N \right) \left(\frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*) h(\hat{\theta}^*)} + 1 \right) \frac{1}{\lambda^* \cdot (1 - F(\hat{\theta}^*))} - \frac{\nu^N}{\lambda^*}. \tag{87}$$

Observe that, as desired, this expression is positive if $f'(\hat{\theta}^*) \geq 0$, because $\lambda^* \cdot (1 - F(\hat{\theta}^*)) < 1$.

It remains to show part (iii). It suffices to show that (87) is positive as search costs get small. Recall that by Footnote 23, effective search costs σ^* vanish as $s \to 0$, so that $\hat{\theta}^* \to \overline{\theta}$ as $s \to 0$ by Lemma A.1. We hence find that

$$\lim_{s \to 0} \left(\frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*) h(\hat{\theta}^*)} + 1 \right) \frac{1}{(1 - F(\hat{\theta}^*))} = \lim_{\hat{\theta}^* \to \overline{\theta}} \left(\frac{f'(\hat{\theta}^*)}{f(\hat{\theta}^*) h(\hat{\theta}^*)} + 1 \right) \frac{1}{(1 - F(\hat{\theta}^*))}. \tag{88}$$

Notice that (87) is positive as search costs get small if (88) diverges. The reason why (88) diverges is that, on the one hand, f'/f is bounded from below,[40] while the hazard rate diverges by assumption, and on the other hand, $1 - F(\hat{\theta}^*)$ tends to zero as $\hat{\theta}^* \to \overline{\theta}$. This completes the proof. ■

Proof of Lemma 3 We have to show that

$$\frac{d\sigma^*}{ds} \geq 0. \tag{89}$$

[40]By log-concavity of f, f'/f is decreasing and $f'(\overline{\theta})/f(\overline{\theta})$ is bounded from below by (23) and $f(\overline{\theta}) > 0$.

Indeed, let $\sigma = s/\lambda$, and define the function

$$\tilde{\Delta}(\sigma) = \Delta\left(\frac{s}{\sigma}\right) = \left(\nu^S \cdot \frac{\sigma}{s} + \nu^N\right) \cdot \frac{1}{h(g^{-1}(\sigma))} - \nu^N(\sigma + \Delta c). \tag{90}$$

The equilibrium condition $\Delta(\lambda^*) = 0$ implies that $\tilde{\Delta}(\sigma^*) = 0$, and hence

$$\frac{d\sigma^*}{ds} = -\frac{\partial \tilde{\Delta}/\partial s}{\partial \tilde{\Delta}/\partial \sigma}. \tag{91}$$

Differentiating (90) yields that

$$\frac{\partial \tilde{\Delta}(\sigma^*)}{\partial \sigma} = \frac{\partial \Delta(\lambda^*)}{\partial \lambda}\frac{\partial \lambda}{\partial \sigma} = \frac{\partial \Delta(\lambda^*)}{\partial \lambda}\left(-\frac{s}{\sigma^*}\right), \tag{92}$$

and $\partial \tilde{\Delta}(\sigma)/\partial \sigma$ is thus positive, because $\partial \Delta(\lambda^*)/\partial \lambda < 0$ by (70). Moreover, by (90):

$$\frac{\partial \tilde{\Delta}}{\partial s} = -\nu^S \frac{\sigma^*}{s^2} \cdot \frac{1}{h(g^{-1}(\sigma^*))}. \tag{93}$$

Because this is negative, it follows that $d\sigma^*/ds$ is positive, as desired. ■

Proof of Proposition 5 The argument is given in the main text. ■

Proof of Proposition 6 As to (i), by (43),

$$\frac{dW}{ds} = \nu^S \cdot \frac{d\hat{\theta}^*}{ds} + \nu^N \cdot \frac{d\lambda^*}{ds} \cdot (\Delta q - \Delta c) - \nu^N. \tag{94}$$

By (24), we have $g(\hat{\theta}^*) = \sigma^*$, and hence

$$\frac{d\hat{\theta}^*}{ds} = \frac{d\sigma^*}{ds}\frac{1}{g'(\hat{\theta}^*)}, \tag{95}$$

which is negative, because $g' = -(1-F)$ by Lemma A.1 and $d\sigma^*/ds \geq 0$ by Lemma 3. Moreover, by assumption $d\lambda^*/ds \leq 0$ and $\Delta q - \Delta c \geq 0$. Together this implies that (94) is negative.

To see part (ii), observe that if $d\lambda^*/ds > 0$, then (94) is positive if $\Delta q - \Delta c \geq 0$ is sufficiently large. This completes the proof. ■

Proof of Proposition 7 Recall that by (7), sophisticated consumer welfare is $\hat{U}^S = \overline{q} + \hat{\theta}^* - \overline{p}^*$. As argued after (95), $\hat{\theta}^*$ decreases in s. Moreover, $\overline{p}^*$ increases in s by Proposition 5, because λ^* increases in s by assumption. This implies that $\hat{U}^S$ indeed decreases in s.

As to profits, recall that deceptive profits are given by $\underline{\pi}^* = \nu^N \cdot (\underline{p}^* - \underline{c})$. Because λ^* increases in s by assumption, deceptive prices increase in s by Proposition 5. Therefore, deceptive profits increase in s, as desired.

Finally, we obtain from (13):

$$\hat{U}^N = \underline{q} + \lambda^* \cdot \Delta q) - \lambda^* \cdot \overline{p}^* - (1-\lambda^*) \cdot \underline{p}^* - s. \tag{96}$$

Because $\overline{p}^* = \underline{p}^* - s/\lambda^*$ by (29), this simplifies to

$$\hat{U}^N = \underline{q} + \lambda^* \cdot \Delta q - \underline{p}^*. \tag{97}$$

If λ^* increases in s, the second term increases in s, whereas the third term decrease in s, because deceptive price increase in s if λ^* increases in s by Proposition 5. Hence, the overall effect is not clear–cut. This completes the proof. ■

Proof of Proposition 5 As to (i). Because $d\lambda^*/d\nu^N = -(\partial\Delta/\partial\nu^N)/(\partial\Delta/\partial\lambda)$ and $\partial\Delta/\partial\lambda < 0$ by (70), it is sufficient to show that

$$\frac{\partial\Delta}{\partial\nu^N} < 0. \tag{98}$$

Recall that $\Delta = (\nu^N + \nu^S/\lambda^*) \cdot (\overline{p}^* - \overline{c}) - \nu^N \cdot (\overline{p}^* + \sigma^* - \underline{c})$. Hence,

$$\begin{aligned}
\frac{\partial\Delta}{\partial\nu^N} &= \left(1 - \frac{1}{\lambda^*}\right) \cdot (\overline{p}^* - \overline{c}) + \left(\nu^N + \frac{\nu^S}{\lambda^*}\right) \cdot \frac{\partial\overline{p}^*}{\partial\nu^N} - (\overline{p}^* + \sigma^* - \underline{c}) - \nu^N \cdot \frac{\partial\overline{p}^*}{\partial\nu^N} && (99)\\
&= \left(1 - \frac{1}{\lambda^*}\right) \cdot (\overline{p}^* - \overline{c}) - (\overline{p}^* + \sigma^* - \underline{c}) + \frac{\nu^S}{\lambda^*} \cdot \frac{\partial\overline{p}^*}{\partial\nu^N} && (100)\\
&= -\frac{1}{\lambda^*} \cdot \frac{1}{\nu^N} \cdot (\overline{p}^* - \overline{c}) + \frac{\nu^S}{\lambda^*} \cdot \frac{\partial\overline{p}^*}{\partial\nu^N}, && (101)
\end{aligned}$$

where we have inserted the equilibrium condition $\Delta(\lambda^*) = 0$ (which is equivalent to $\overline{p} + \sigma^* - \underline{c} = 1/\nu^N \cdot (\nu^N + \nu^S/\lambda^*) \cdot (\overline{p}^* - \overline{c})$) in the second line to obtain the third line. Recall from (34) that

$$\overline{p}^* - \overline{c} = \left(1 + \lambda^* \frac{\nu^N}{\nu^S}\right) \cdot \frac{1}{h(\hat{\theta}^*)}, \tag{102}$$

and observe that

$$\frac{\partial\overline{p}^*}{\partial\nu^N} = \frac{\lambda^*}{(1-\nu^N)^2} \cdot \frac{1}{h(\hat{\theta}^*)}. \tag{103}$$

After inserting (102) and (103) in (101) and simplifying terms, we obtain

$$\frac{\partial\Delta}{\partial\nu^N} = -\frac{1}{\lambda^* h(\hat{\theta}^*)\nu^N}, \tag{104}$$

which establishes (98) and completes the proof of part (i).

As to (ii). As argued in the text, it suffices to show that

$$\frac{\partial \overline{p}^*}{\partial \lambda} + \frac{\partial \overline{p}^*}{\partial \sigma} \cdot \frac{\partial \sigma^*}{\partial \lambda} \quad \leq \quad 0 \quad \text{if} \quad \epsilon(\sigma^*) \geq 1 \tag{105}$$

Too see this, observe that (34), (35) and a straightforward calculation yield that

$$\frac{\partial \overline{p}^*}{\partial \lambda} + \frac{\partial \overline{p}^*}{\partial \sigma} \cdot \frac{\partial \sigma^*}{\partial \lambda} \quad = \quad \frac{\nu^N}{\nu^S} \cdot \frac{1}{h(\hat{\theta}^*)} + \epsilon(\sigma^*) \cdot \frac{(\overline{p}^* - \overline{c})}{\sigma^*} \cdot (-\frac{s}{(\lambda^*)^2}). \tag{106}$$

Inserting (34) and simplifying terms yields that

$$\frac{\partial \overline{p}^*}{\partial \lambda} + \frac{\partial \overline{p}^*}{\partial \sigma} \cdot \frac{\partial \sigma^*}{\partial \lambda} \quad = \quad \left(\frac{\nu^N}{\nu^S} - \epsilon(\sigma^*) \cdot (\frac{1}{\lambda^*} + \frac{\nu^N}{\nu^S}) \right) \cdot \frac{1}{h(\hat{\theta}^*)}, \tag{107}$$

which establishes (105), as desired. ■

Proof of Proposition 9 As to (i). Consider first sophisticated consumer welfare. By (7),

$$\frac{dU^S}{d\nu^N} = \frac{d\theta^*}{d\nu^N} - \frac{d\overline{p}^*}{d\nu^N}. \tag{108}$$

To see that (108) is negative, as desired, observe first that $d\overline{p}^*/d\nu^N > 0$ by Proposition 5 and second that $d\hat{\theta}^*/d\nu^N < 0$, because by (10),

$$\frac{d\theta^*}{d\nu^N} = \frac{\partial \theta^*}{\partial \lambda} \cdot \frac{d\lambda^*}{d\nu^N} = -\frac{s}{\lambda^{*2} g'(\hat{\theta}^*)} \cdot \frac{d\lambda^*}{d\nu^N} \leq 0, \tag{109}$$

where the inequality follows from $g'(\hat{\theta}^*) < 0$ by Lemma A.1 and $d\lambda^*/d\nu^N \leq 0$ by Proposition 5.

Next, we consider naive consumer welfare. Recall from (97) that $U^N = \underline{q} + \lambda^* \cdot \Delta q - \underline{p}^*$. By Proposition 5, λ^* decreases in ν^N and $\underline{p}^*$ increases in ν^N. Thus, U^N decreases in ν^N.

Finally, consider firm profits. Recall that a deceptive firm's profit is given by $\underline{\pi} = \nu^N \cdot (\underline{p}^* - \underline{c})$, and thus increases in ν^N, because $\underline{p}^*$ increases in ν^N by Proposition 5.

As to (ii). By (43), a straightforward calculation delivers that

$$\begin{aligned} \frac{dW}{d\nu^N} &= -(\overline{q} - \overline{c} + \hat{\theta}^*) + \nu^S \cdot \frac{d\hat{\theta}^*}{d\nu^N} + \lambda^* \cdot (\Delta q - \Delta c) + \underline{q} - \underline{c} - s + \nu^N \cdot \frac{d\lambda^*}{d\nu^N} \cdot (\Delta q - \Delta c) && (110) \\ &\leq -(1 - \lambda^*) \cdot (\Delta q - \Delta c) - \hat{\theta}^* - s, && (111) \end{aligned}$$

where the inequality follows from $d\hat{\theta}^*/d\nu^N \leq 0$ by (109), $d\lambda^*/d\nu^N \leq 0$ by Proposition 5, and re–arranging terms.

Now, a revealed preference argument implies that the equilibrium utility of a sophisticated consumer, $U^S = \overline{q} - \overline{p}^* + \hat{\theta}^*$, is larger than the expected utility that he would obtain if he purchased the first candid product that he encounters, which is $\overline{q}-\overline{p}^*+\mathbb{E}(\theta)-\sigma^*$. Because $\mathbb{E}(\theta)=0$, this implies that $\sigma^*\geq-\hat{\theta}^*$. Inserting this in (111) yields (recall $s=\sigma^*\cdot\lambda^*$)

$$\frac{dW}{d\,\nu^N} \leq -(1-\lambda^*)\cdot(\Delta q-\Delta c+\hat{\theta}^*), \tag{112}$$

and $\Delta q-\Delta c\geq-\hat{\theta}^*$ thus implies $dW/d\,\nu^N<0$. Finally, $\Delta q-\Delta c\geq-\hat{\theta}^*$ follows from the two observations that $\Delta q-\Delta c\geq\overline{\theta}-\underline{\theta}$ (because $\hat{\theta}^*\geq\underline{\theta}$), and that $\Delta q-\Delta c\geq\sigma^*$ (because $\sigma^*\geq-\hat{\theta}^*$ as argued above). ■

Proof of Lemma 4 The argument why a price floor improves total welfare is given in the main text.

As to consumer welfare, observe that by (7), (29) and (97) consumer welfare is given as

$$W^C = \nu^S\cdot(\overline{q}+\hat{\theta}-\overline{p})+\nu^N\cdot(\underline{q}+\lambda\cdot\Delta q-(\overline{p}+\frac{s}{\lambda})), \tag{113}$$

where $\hat{\theta}$ and λ are the equilibrium market outcomes given a candid price $\overline{p}$ and are pinned down by (24) and (47). We want to show that

$$\frac{dW^C}{d\overline{p}}>0 \quad \text{if} \quad \Delta q \text{ is sufficiently large.} \tag{114}$$

By (24) and Lemma A.1, $\hat{\theta}$ increases in λ, and by (47), λ increases in $\overline{p}$. Therefore, $\hat{\theta}$ increases in $\overline{p}$, and we infer that

$$\frac{dW^C}{d\overline{p}} \geq -\nu^S+\nu^N\cdot(\frac{d\lambda}{d\overline{p}}\cdot\Delta q-1). \tag{115}$$

Differentiating (47) yields an expression for $d\lambda/d\overline{p}$, and, after simplifying terms, we obtain

$$\frac{dW^C}{d\overline{p}} \geq -1+\frac{\nu^S}{\Delta c}\cdot\Delta q, \tag{116}$$

which is positive if Δq is sufficiently large, as desired. ■

Proof of Lemma 5 As to total welfare, as argued in the text, it suffices to show that the share of candid firms increases in response to a minimum quality standard. To see this, observe that the difference in candid and deceptive profits as given in (31) is independent of $\underline{q}$. The effect of a minimum quality standard on the share of candid firms is therefore given by $d\lambda^*/d\underline{c} = -(\partial\Delta/\partial\underline{c})/(\partial\Delta/\partial\lambda)$. Because $\partial\Delta/\partial\lambda<0$ by (70),

$$\frac{\partial\Delta}{\partial\underline{c}} = \nu^N \tag{117}$$

implies $d\lambda^*/d\underline{c} \geq 0$, as desired.

As to consumer welfare, as is argued in the text, if suffices to show that $d\overline{p}^*/d\lambda \leq 0$ if $\epsilon(\sigma^*) \geq 1$. But this follows immediately from (105). ∎

Proof of Lemma 6 First, we show that there is no pooling equilibrium. To the contrary, suppose that there is a segmented market equilibrium such that all firms charge p^*, and the share of candid firms is $\lambda^* \in (0,1)$. In a pooling equilibrium, an uninformed consumer cannot distinguish between candid and deceptive products and purchases from the first firm he visits in equilibrium. Because a consumer's reservation utility equals his expected equilibrium utility, the reservation utility of an uninformed consumer is therefore given by

$$\hat{U}^N = \lambda^* \cdot \overline{q} + (1-\lambda^*) \cdot \underline{q} - p^* - s. \tag{118}$$

Now, an uninformed consumer expects any product offered at p to supply the utility

$$U(p) = \hat{\lambda}(p) \cdot \overline{q} + (1-\hat{\lambda}(p)) \cdot \underline{q} - p. \tag{119}$$

Therefore, he purchases a deceptive product offered at $p^* + dp$, because $U(p^* + dp) > \hat{U}^N$ for small dp by consistency and continuity of $\hat{\lambda}$. As a deceptive firm only derives demand from uninformed consumers in a segmented market equilibrium, it would be profitable for it to raise its price marginally – a contradiction of the optimality of prices in equilibrium.

Second, we show that there is no separating equilibrium. Suppose the contrary and consider first the case that the price of the deceptive product exceeds the price of the candid product, that is, $\underline{p}^* > \overline{p}^*$. Because in a separating equilibrium uninformed consumers recognize deceptive products as what they are (on the equilibrium path), for a deceptive firm to derive demand from uninformed consumers in equilibrium, uninformed consumers must prefer purchasing a low quality product at $\underline{p}^*$ to searching for a candid product:

$$(\underline{p}^* - \overline{p}^*) + \Delta q < \frac{s}{\lambda^*}. \tag{120}$$

To see the equation, note that s/λ^* are the expected costs to find a candid product not only for a sophisticated consumer but also for an uninformed consumer, as he recognices candid products as such in a separating equilibrium. To continue, (120) implies $\Delta q < s/\lambda^*$ because $\underline{p}^* > \overline{p}^*$ by assumption. By assumption (4), we thus have $\overline{\theta} - \underline{\theta} < s/\lambda^*$. Now, in a segmented market equilibrium a sophisticated consumer only purchases candid products and, as before, his search rule is characterized by a reservation match-value $\hat{\theta}^*$ that is given by (10). By (10) and lemma A.1, $\overline{\theta} - \underline{\theta} < s/\lambda^*$ implies $\hat{\theta} < \underline{\theta}$. In equilibrium, a sophisticated consumer has therefore strict incentives to purchase a candid product at $\overline{p}^*$, irrespective of product fit. Note, however, that also uninformed consumers must have strict incentives to purchase candid products at $\overline{p}^*$, because they purchase in equilibrium the deceptive product which is more expensive and provides a lower quality. Thus, the entire demand of a candid firm is (locally) inelastic at $\overline{p}^*$ (by continuity of $\hat{\lambda}$) such that it is profitable for a candid firm to increase its price – contradicting that $\overline{p}^*$ is an equilibrium price.

Next consider the case that $\underline{p}^* < \overline{p}^*$ holds. Because an uninformed consumer purchases candid products in a segmented market equilibrium, he purchase any product that he encounters and that is offered at $\overline{p}^*$ with strictly positive probability $\xi \in (0,1]$. If $\xi = 1$, then a deceptive firm could charge $\overline{p}^*$ and any visiting uninformed consumers would purchase its product. Because $\underline{p}^* < \overline{p}^*$ and because a deceptive firm only derives demand from uninformed consumers in a segmented market equilibrium, this would constitute a profitable deviation – a contradiction. Therefore, $\xi \in (0,1)$ and an uninformed consumer is indifferent about whether to purchase a product offered at $\overline{p}^*$. More precisely, an uninformed consumers is indifferent about whether to purchase a candid product offered at $\overline{p}^*$, because due to consistency he expects a firm to provide the candid quality if it charges $\overline{p}^*$. We thus have

$$\hat{U}^N = \overline{q} - \overline{p}^*. \tag{121}$$

Because a naive consumer purchases candid and deceptive products in equilibrium, it is optimal for him to purchase at the first firm he visits. His reservation utility therefore satisfies

$$\hat{U}^N = \lambda^* \cdot (\overline{q} - \overline{p}^*) + (1-\lambda^*) \cdot (\underline{q} - \underline{p}^*) - s, \tag{122}$$

and by (121), we have

$$\hat{U}^N = (\underline{q} - \underline{p}^*) - \frac{s}{1-\lambda^*}. \tag{123}$$

By consistency of $\hat{\lambda}$, an uninformed consumer expects a product offered at $\underline{p}^*$ to supply the utility

$$U(\underline{p}^*) = \underline{q} - \underline{p}^*. \tag{124}$$

By (123) and (124), an uninformed consumer has therefore strict incentives to purchase a product offered at $\underline{p}^*$. As a consequence, a deceptive firm could raise its price marginally and would not loose any demand from uninformed consumers (by continuity of $\hat{\lambda}$). As a deceptive firm only derives demand from uninformed consumers in a segmented market equilibrium, this contradicts the optimality of prices in equilibrium. ■

Proof of Lemma 7 First, we argue that there is a segmented market equilibrium with $\underline{p}^* > \overline{p}^*$ if ε and search costs s are sufficiently small. Second, we argue that the share of candid firms vanishes as search costs vanish.

Before, observe that simple algebra yields that in a segmented market equilibrium the expected quality of a product given a good respectively bad signal is

$$\mathbb{E}(q|\overline{s}) = \frac{\lambda(\frac{1}{2}+\varepsilon)\overline{q}+(1-\lambda)(\frac{1}{2}-\varepsilon)\underline{q}}{\lambda(\frac{1}{2}+\varepsilon)+(1-\lambda)(\frac{1}{2}-\varepsilon)}, \quad \text{respectively} \quad \mathbb{E}(q|\underline{s}) = \frac{\lambda(\frac{1}{2}-\varepsilon)\overline{q}+(1-\lambda)(\frac{1}{2}+\varepsilon)\underline{q}}{\lambda(\frac{1}{2}-\varepsilon)+(1-\lambda)(\frac{1}{2}+\varepsilon)}.$$

Observe that $\mathbb{E}(q|\overline{s}) \geq \mathbb{E}(q) \geq \mathbb{E}(q|\underline{s})$ and for each δ there exists ε such that $\mathbb{E}(q|\overline{s}) - \mathbb{E}(q|\underline{s}) < \delta$.

Claim *For sufficiently small ε and sufficiently small search costs s, there is a solution $\overline{p}^* \geq 0$, $\underline{p}^* \geq 0$, $\hat{p}^* \geq 0$, $\hat{\theta}^* \in (\underline{\theta}, \overline{\theta})$ and $\lambda^* \in (0,1)$ to the equation system (24), (26-28) and*

$$\hat{p}^* = \lambda^* \cdot \overline{p}^* + (1-\lambda^*) \cdot \underline{p}^* + s - (\mathbb{E}(q) - \mathbb{E}(q|\underline{s})) \tag{125}$$

with $\underline{p}^* > \overline{p}^*$ and this solution describes a segmented market equilibrium outcome.

Proof First, we show that the above equation system indeed characterizes the best responses of all agents in expectation of a segmented market outcome that solves the above equation system.

With regard to (125), (27) and (125) imply

$$\underline{p}^* \quad = \quad \overline{p}^* + \frac{s - (\mathbb{E}(q) - \mathbb{E}(q|\underline{s}))}{\lambda^*}. \tag{126}$$

For sufficiently small ε, we have $s > \mathbb{E}(q) - \mathbb{E}(q|\underline{s})$, and thus $\overline{p}^* < \underline{p}^*$. Moreover, by (27), we have $\underline{p}^* = \hat{p}^*$. Together, this implies $\overline{p}^* < \underline{p}^* \leq \hat{p}^*$. Now, if it is optimal for a naive consumer to purchase at the first firm he visits (even if he receives a bad signal), then his reservation utility is given by (13). In this case, (125) is a complete characterization of a naive consumer's optimal search rule (his reservation utility), because (125) denotes the maximal price that he is willed to pay for a product upon receiving a bad signal given (13), as can be shown straightforwardly. To see that it is indeed optimal for a naive consumer to purchase at the first firm he visits, it suffices to verify that $\overline{p}^* < \underline{p}^* \leq \hat{p}^*$ holds, which is the case, as was shown before.

With regard to (24), as argued in the proof of Lemma 2, $\underline{p}^* > \overline{p}^*$ implies that a sophisticated consumer does not purchase deceptive products and (24) characterizes his optimal search rule, as before.

As to (26), for prices below $\hat{p}^*$, profits of candid firms are given by (18). Because $\underline{p}^* > \overline{p}^*$ and $\underline{p}^* = \hat{p}^*$, (18) is differentiable at $\overline{p}^*$ and (26) is the first order condition for profit maximization of candid firms, as before.

As to (27), for a deceptive firm it is optimal to charge $\hat{p}$, because its demand from naive consumer is inelastic up to $\hat{p}$ (leaving the demand from sophisticated consumer aside). If a deceptive firm charged a price that exceeds $\hat{p}$, it would only derive demand from naive consumer who receive a good signal about its quality. At most a deceptive firm could raise its price (and hence its mark–up) by $\mathbb{E}(q|\overline{s}) - \mathbb{E}(q|\underline{s})$, so to still derive demand from these consumers, whereas it would loose the demand of all naive consumers that receive a bad signal which amounts $\nu^N \cdot (1/2 - \varepsilon)$. For ε sufficiently small, a deceptive firm hence looses almost half of its demand, whereas it only increases its mark–up by less than some δ. Because $\underline{p}^* > \overline{p}^*$, the mark–up of a candid firm is bounded away from zero in equilibrium, which implies that such a deviation is not profitable for sufficiently small ε.

Finally, (28) ensures that firms in the two segments earn the same profits.

A proof why the first order conditions is sufficient for profit maximization for a candid firm and why it not profitable for a deceptive firm to lower its price by so much that it derives demand from sophisticated consumers follows along the proof of Lemma 2. Moreover, an argument why there is a solution to the above the equation system for sufficiently small search costs follows along the lines of Proposition 2. We omit these details. □

We are now in position to show that the share of candid firms vanishes as search costs do. Because $\underline{p}^* > \overline{p}^*$, the deceptive mark–up is bounded away from zero. This implies that deceptive profits are bounded away from zero, because the demand of a deceptive firm is ν^N. Now, suppose that λ^* would not vanish as search costs get small. Then, λ^* would be bounded from below and effective search costs would vanish as search costs get small. This would imply that a candid firm's mark–up and profits vanish, because, as before, a candid firm's price is given by (26). This contradicts that firms in the two segments earn the same profit. ■

B Appendix

In this appendix, we show that if the distribution f of match-values is truncated exponential and given by (39), then there an open set of parameters $s, \overline{q}, \underline{q}, \underline{c}, \overline{c}, \nu^N$ such that there is a segmented market equilibrium with $\hat{\theta}^* < 0$ and

$$\lambda^* = \frac{\frac{\nu^S}{\nu^N} - s}{\Delta c - 1} \quad \text{and} \quad \overline{p}^* - \overline{c} = \frac{\Delta c - s\frac{\nu^N}{\nu^s}}{\Delta c - 1}. \tag{127}$$

Note that the cumulative distribution function of the truncated exponential is

$$F^E(\theta) = \begin{cases} 1 - e^{-(\theta+1)} & \theta < 0 \\ 1 + e^{-1} \cdot (\theta - 1) & \theta \geq 0 \end{cases} \tag{128}$$

and satisfies $h^E(\theta) = 1$ for $\theta \leq 0$.

Moreover, the corresponding function g^E as defined by (9) is given by[41]

$$g^E(z) = \begin{cases} \frac{1}{2e} \cdot (2e^{-z} - 1) & z < 0 \\ \frac{1}{2e} \cdot (1-z)^2 & z \geq 0. \end{cases} \tag{132}$$

We now show that provided $\hat{\theta}^* < 0$, the formulae in (127) satisfy the equilibrium conditions

[41]To see this, note that (9) implies for $z < 0$:

$$g^E(z) = \int_z^0 (\theta - z)\, e^{-(\theta+1)}\, d\theta + \int_0^1 (\theta - z)\, e^{-1}\, d\theta. \tag{129}$$

Integration by parts yields

$$\int_z^0 (\theta - z)\, e^{-(\theta+1)}\, d\theta \quad = \quad -(\theta - z)\, e^{-(\theta+1)}\Big|_z^0 + \int_z^0 e^{-(\theta+1)}\, d\theta = (z-1)\, e^{-1} + e^{-(z+1)}. \tag{130}$$

Moreover, $\int_0^1 (\theta - z)\, e^{-1}\, d\theta = e^{-1} \cdot (\frac{1}{2} - z)$. Inserting this in (129) yields the first line in (132.)

For $z \geq 0$, g^E corresponds to the second line in (132), because by (9):

$$g^E(z) = \int_z^1 (\theta - z)\, e^{-1}\, d\theta = e^{-1} \cdot (\frac{1}{2}\theta^2 - z\theta)\Big|_z^1 = e^{-1} \cdot (\frac{1}{2} - z + \frac{1}{2}z^2) = \frac{1}{2e} \cdot (1-z)^2. \tag{131}$$

(26-28).[42] In fact, $\hat{\theta}^* < 0$ implies $h^E(\hat{\theta}^*) = 1$, and thus $\Delta(\lambda^*) = 0$ becomes equivalent to

$$\frac{\nu^S}{\lambda^*} \cdot (1 + \lambda^* \frac{\nu^N}{\nu^S}) - \nu^N \cdot (\frac{s}{\lambda^*} + \Delta c) = 0. \tag{133}$$

Rearranging terms then yields the desired expression for λ^*. Inserting $h^E(\theta^*) = 1$ and λ^*, as given in in (127), in equation (26) yields

$$\overline{p}^* - \overline{c} = 1 + \frac{1 - s\frac{\nu^N}{\nu^S}}{\Delta c - 1}), \tag{134}$$

which is equivalent to the expression in (127).

To complete the proof, we show that there are parameters so that for λ^* as given in (127), we have that $\lambda^* \in (0, 1)$ and the equilibrium match-value $\hat{\theta}^*$ which is given as $\hat{\theta}^* = (g^E)^{-1}(s/\lambda^*)$ by (24) indeed satisfies $\hat{\theta}^* < 0$.

To see this, let $\nu^S/\nu^N = 1/e$, $s = 1/(2e)$ and $\Delta c = 1 + 1/e$. By a straightforward but tedious calculation, we obtain

$$\lambda^* = \frac{1}{2}, \quad \hat{\theta}^* = -\ln(3/2), \quad \overline{p}^* = \frac{e}{2} + 1, \quad \underline{p}^* = \frac{e}{2} + 1 + \frac{1}{e}. \tag{135}$$

Finally, note that because the equilibrium values are continuous in the parameters, if we perturb the parameters slightly, this does not upset the (strict) inequalities $\lambda^* \in (0, 1)$ and $\hat{\theta}^* < 0$. This establishes the claim. ∎

[42] Strictly speaking, it is not a priori clear that with the truncated exponential distribution, the equilibrium conditions (24-28) are actually sufficient for equilibrium existence. The reason is that f^E is not globally log-concave (which we use in our sufficiency proof). However, for the specific case that $f = f^E$, the sufficiency proof still goes through with minor changes. We omit the details.

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